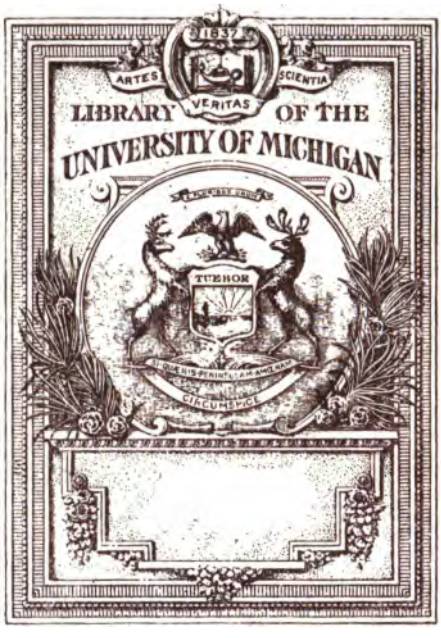
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ACTUARIAL STUDIES NO. 5

TOTAL PERMANENT DISABILITY BENEFITS IN RELATION TO LIFE INSURANCE

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GENERAL INTRODUCTION.

In view of the fact that, with the exception of a very few modern text books, the literature of Actuarial Science is contained in scattered original papers, The Actuarial Society of America proposes to issue a series of small volumes upon important actuarial subjects. Each volume is intended to bring together, as far as space permits, the more important points of information on the subject discussed. The objects in issuing the series are twofold: (1) to assist students of Actuarial Science, and (2) to furnish a means of ready reference for Actuaries. The various subjects are allocated to Fellows of the Society by the Committee in Charge; and, associated with the principal contributor, who is primarily responsible for the matter included and the views expressed, are one or more "Associate Contributors." These are appointed for the purpose of aiding and criticizing the work before publication. It is proposed to avoid discussing subjects already covered in the Text Book of the Institute of Actuaries except as continuity of thought may make occasional references necessary. The title chosen to represent the character of this series is "Actuarial Studies."

The thanks of the Society and of the Committee in Charge are due to all the contributors who have freely given of their time and labor, with the sole purpose of helping others—especially students.

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PREFACE.

The first draft of this study was completed more than six years ago. As there has been such a great liberalization and expansion in the last three years in Permanent Total Disability Benefits it is fortunate that the study was not then published. During 1918 it was accordingly necessary to rewrite this study and to extend it greatly. The associate contributors for the original study were Franklin B. Mead and Sidney H. Pipe. Owing to warfare conditions they have not been able to help with the revision.

Arthur Coburn, William Macfarlane and Adolph A. Rydgren have rendered valuable services in the preparation of this study.

As few companies, none of large size, had any form of disability benefit in their policies prior to 1910 the amount of experience on this valuable adjunct to life insurance is limited. It is accordingly recognized that some of the information in this study may be found to be out-of-date in a few years, if the companies give their experience to the actuarial profession.

A. H.

15th April, 1920.

CONTENTS.

TOTAL PERMANENT DISABILITY BENEFITS IN RELATION TO LIFE INSURANCE.

This study deals with benefits to the insured in the event of his becoming totally and permanently disabled, and does not consider total disability of a temporary nature or partial disability of a permanent kind. As examples of these three we might consider incurable insanity as total and permanent disability, typhoid fever as total disability of a temporary nature, and the loss of a hand as partial disability of a permanent nature. Provisions regarding the first mentioned are frequently made a part of life insurance policies, and are not usually covered by separate contracts. The first policies containing this form of insurance appear to have been granted in Germany in 1876 and by American fraternal orders in the succeeding year, but were not adopted by any regular life insurance company in the United States until 1896, and not generally until 1907 and thereafter. Germany has been the principal exponent of this feature, a majority of the companies granting it in one form or another. In Russia a number of companies grant it, while in Britain a very few companies make use of the feature. It is not the intention to give a history of disability in the various countries in Europe, because the conditions are rapidly changing and a statement of the facts at this time would probably be of little value. References to the history may be found in the contributions to the *Transactions of the Actuarial Society* by C. W. Jackson (X, 490), and F. B. Mead (XI, 304). We shall have occasion to refer to these papers in other connections and also to those of S. H. Pipe (XI, 172 and XIII, 20), Arthur Hunter (XII, 44 and XIX, 219), F. B. Mead (XII, 75), E. B. Fackler (XII, 241), J. F. Little (XIV, 55), G. Bohlmann (XV, 77), E. B. Morris (XV, 98), Mervyn Davis (XVII, 211) and J. B. Gibb (XVIII, 122). The notation and formulæ were first developed by E. Hamza in a paper delivered before The Third International Congress of Actuaries (p. 154 of the Transactions).

Hereafter, when the word "disability" is used, it will cover total and permanent disability unless otherwise specified.

DISABILITY PROVISIONS IN LIFE INSURANCE POLICIES.

Principal Types of Disability Benefits.

The following are the principal types of disability benefits appearing in the policies of life insurance companies:

1. Waiver of future premiums only.

2. Waiver of premiums and payment of the sum insured in equal instalments over a period of 10, 15 or 20 years with the balance of the insurance payable in one sum if death occur before all the instalments have been paid.

3. Waiver of premium and payment of an annuity, usually 10 per cent. annually or 1 per cent. monthly, of the sum insured, during the lifetime of the insured while disabled or to the maturity of the contract, and payment of the face of the policy at maturity or death.

With regard to 2, when disability benefits were introduced in America, a number of companies granted a somewhat similar benefit, the insured upon disablement being required to select one of two options:

(*a*) Waiver of premiums during permanent and total disability, the sum insured to be paid in full at death or maturity; or
(*b*) The policy to mature at once and the proceeds to be payable in instalments for 10, 15, or 20 years, the aggregate amount of such instalments being the sum insured.

The values of these options at disability vary in relation to the death rate amongst disabled lives. If the insured upon disablement selected Option (*b*) and died soon after, the beneficiary would only get the instalments or their present value, whereas under Option (*a*) the full sum insured would be payable; consequently Option (*b*) led to dissatisfaction. There is no proper basis for the calculation of premiums and reserves for this type.

With regard to (3), if the proportion of the sum insured which is payable as an annuity be taken as d, the rate of discount, it follows that the benefit is equivalent to maturing the policy, because provision is made for interest in advance annually to death or end of endowment period. Maturity of the policy therefore appears as a particular case of benefit (3), and the formulae developed for (3) may be applied by taking an annuity of d times the sum insured and considering it payable annually

in advance from the date provided for maturity, generally six months after the filing of proof of disability. It should be noted, however, that the case of term insurance furnishes an exception. Here the loss to the company at the disability of the insured must also comprise the pure endowment required at that time to mature the policy at the end of the term as that is not provided in the original contract.

There are many variations of the three foregoing principal types, and a number of special types, but these are generally confined to single companies. Several companies in Europe agree that in the event of the insured being totally incapacitated for two or more months, they will waive such proportion of the next premium as the period of total incapacity bears to the period covered by the premium. This may include either temporary or permanent disability, and so far as it partakes of the nature of the former will not be considered in this study.

With few exceptions the wording of the clauses in the policies covers the inability to follow *any* business or profession, because a slight injury might prevent carrying on a particular occupation, but permit engaging in an equally advantageous business. This does not mean that the companies rigidly refuse to allow the benefit because *some* occupation could be followed, if the insured fitted himself for it, but is intended to prevent a claim under inability to follow one occupation when the insured has changed to another, which he can follow successfully. In Europe where there are much greater difficulties in the way of changing one's vocation, the policies occasionally specify inability to follow the occupation in which the insured was engaged at the time disability occurred. While this would increase only slightly the cost to the company in Europe, it would be of greater moment in the United States or Canada, and more likely to cause controversy with the insured.

It is customary to provide in the policies that the full benefit will accrue only if the disability occur before a certain age, usually 60 or 65. This is advisable, not only because the rate of disability due to disease increases greatly at the higher ages, but because of the difficulty of differentiating between sickness and the effects of old age.

Examples are now given of clauses covering the three foregoing types of disability benefits.

Type No. 1. Waiver of Premiums.

The premium payable by the terms of this policy includes an additional premium of ..... dollars, payable for ..... full years, or until the prior death of the insured, and in consideration of the payment of such additional premium the ..... Life Insurance Company hereby agrees to the following waiver of premium in the event of permanent total disability. If the insured, after payment of premium for one full year and before default in the payment of any subsequent premium, and before attaining the age of sixty-five years, and while this policy is in full force, shall furnish due proof to the company, at its home office in the city of ....., that he has become wholly disabled by bodily injury or disease so that he is and will be permanently and continuously prevented from performing any work for compensation or profit or from following any gainful occupation, the company will waive payment of each premium as it thereafter becomes due during the continuance of such disability. The premiums so waived shall not be deducted from the sum payable under the policy, and the values provided for under "Non-Forfeiture Provisions" and in the "Table of Loan, Cash, Paid-up and Extended Insurance Values" shall be the same as if the premiums had continued to be paid in cash to the company regularly when due. Provided that, notwithstanding proof of disability may have been accepted by the company as satisfactory, the insured agrees that any medical examiner appointed by the company shall be allowed opportunity, not oftener than once a year thereafter, to satisfy himself by examination of the person of the insured as to the continuance of the disability, and if it shall appear to the company that the insured is able to perform any work or to engage in any occupation whatsoever for compensation or profit, no further premiums will be waived and all premiums thereafter falling due shall be paid by the insured in conformity with the terms of the policy. Irrespective of any other cause of disability the entire and irrecoverable loss of the sight of both eyes, or the severance of both hands at or above the wrists, or of both feet at or above the ankles, or of one entire hand and one entire foot, shall be considered as total and permanent disability within the meaning of this agreement, and the

company upon due proof of such loss or severance will waive the premiums thereafter becoming due.

Type No. 2. Payment of the Face Amount in Instalments, Usually Ten, the Balance of the Face Amount, if Any, Payable at Death or Maturity.

After one full year's premium shall have been paid upon this policy, then if at any time, while no premium is in default, the company shall receive due proof that the insured, prior to attaining the age of sixty years, has become wholly and permanently disabled by bodily injury or by disease sustained or contracted after the date hereof other than as the result during the first policy year of military or naval service in time of war or of making or undertaking an aerial flight, and that thereby he will be wholly, continuously and permanently prevented from the pursuit of any form of mental or manual labor for compensation, gain or profit whatsoever, and has been so disabled for not less than sixty days, the company will waive, after the then current policy year, payment of further premiums during the continuance of the disability, and, at the end of one year from the date on which said disability is proved, provided the disability then exists, will pay to the insured, with the written consent of the assignee, if any, one tenth of the face amount of this policy, and one tenth annually thereafter during the continuance of the disability, until the said face amount, less any indebtedness, shall have been paid in full, or until the maturity of the policy when any unpaid remainder of the sum insured, less any indebtedness, will be paid as provided in the policy. In case there are any accumulations or policy additions from surplus distributions then outstanding they shall be payable with the final payment. The sum insured shall be reduced by each annual instalment so paid and the cash surrender and loan values from year to year will be those shown by the table of non-forfeiture values in the policy, reduced proportionately to the reduced insurance, less any existing indebtedness. Whenever the existing indebtedness shall exceed the cash surrender value of the reduced insurance, the succeeding instalment shall be reduced by the amount of such excess, and the indebtedness correspondingly reduced. Whenever the instalments paid and credited together with the indebtedness shall equal the face

amount of the policy and of any accumulations or additions from surplus distributions, the liability of the company shall cease.

Type No. 3. Payment of an Annuity, Usually 10 Per Cent. Annually or 1 Per Cent. Monthly of the Face Amount, During Disability. Face Amount Payable at Death or at Maturity as an Endowment.

If after one year's premiums shall have been paid on this policy and before default in the payment of any subsequent premium the insured shall furnish to the company due proof that, before attaining the age of sixty, he has become wholly disabled by bodily injury or disease so that he is and thereby will be permanently and continuously unable to engage in any occupation whatever for remuneration or profit, and that such disability has existed continuously for not less than sixty days prior to the furnishing of proof, thereupon the company will grant the following benefits:

A. Waiver of Premium.—The company, by endorsement hereon, shall waive the payment of the premiums which thereafter may become due under this policy during the continuance of the said total disability of the insured. In making any settlement under this policy the company shall not deduct any part of the premiums so waived, and the non-forfeiture values of this policy shall increase from year to year in the same manner as though any premium waived under this provision had been paid in cash.

B. Annuity Payment.—Furthermore, the company will pay to the insured a monthly sum equal to one one-hundredth of the face of this policy, the first monthly payment to be made six months after receipt of due proof of the said total disability accompanied by this policy for endorsement, and subsequent payments monthly thereafter during the continuance of the said total disability of the insured prior to the maturity of this policy. The company will admit the age of the insured when furnished with satisfactory evidence of the date of birth and reserves the right to require such proof of date of birth at the time proof of disability is furnished. The amount of this policy payable at maturity either as an endowment or as a death claim shall

not be reduced by any payments made under this disability provision.

In practically all companies a clause is inserted to the effect that the loss of both eyes, or of both hands, or of both feet, shall be considered as permanent and total disability, although such loss may not seriously hinder a man from carrying on certain occupations. In type No. 1 such a clause appears and a similar clause also appears, although not shown here, in the policies of the companies which issue type No. 2 and type No. 3.

Provision is usually made that the companies shall have the right to call upon the insured for proof of the continuation of the disability, but not oftener than once a year. It can readily be seen that such a clause is necessary both for the protection of the company against continuation of payment of disability benefits when the insured has recovered and for the protection of the insured against having to furnish proof of continued disability too frequently.

The most effective barrier against fraud is a contestable clause for two years as the majority of the fraudulent cases are brought to light within that time.

It will be noted that the disability clause is not made operative on the contraction of any specified disease. The reason is that few diseases are certain to cause total and permanent disability. The experience of life insurance companies indicates that the four principal causes of disability as a result of disease are consumption, insanity, paralysis, and cancer. The insured may recover from any of the first three mentioned, while cancer may not incapacitate from work until a few months before death. There are several diseases which are almost certain to cause total and permanent disability, but these are not so common as the foregoing, and it would not, therefore, be of material value to specify them in the policy.

As each annual premium becomes due the company determines whether the insured is still totally and permanently incapacitated under the terms of the policy. Should he recover his health, the premiums would again become payable but he would not be required to pay the premiums which had been waived. When the company is paying the face of the policy in instalments and such instalments are deducted from the face at death, there is

greater difficulty in adjustment upon the insured's recovery and there does not appear to be uniformity of practice. A prominent company provides in the policy for charging the total amount paid under the disability instalments against the sum insured and considering the balance as the future amount of the insurance under the policy; the premiums would be based on such reduced amount, and the loan, paid-up and other nonforfeiture values would be in the proportion which the face value of the policy, less the total amount of disability instalments paid by the company, bore to the face of the policy. In another company, the disability instalments paid would be considered as a loan at interest and, on the recovery of the insured, would be charged against the policy. The contract would then be in force with the same premium as before the insured became disabled, and the cash, loan, paid-up and term extension values would be the same as for a corresponding policy on which the premiums had been paid regularly. When the disability payments are not deducted from the face of the policy but the face amount is due at death, the practice of most companies, after recovery from disability, is to continue the policy as if premiums had been duly paid and no disability annuity had been paid. The cash, loan, paid-up and extended insurance values would therefore be the same under two policies, of which one had been continued in force by payment of premiums and the other had had a disability annuity paid thereon.

Causes of Disability.

In T. A. S. A., Vol. XIX, p. 219, the experience of three companies as to cause of disability is given. At the time the table was compiled Company "A" had been issuing disability benefit policies for 8 years; Company "B" for 4 years, though this Company, shortly after its adoption of disability benefits, extended these benefits automatically to all policies on its books, without requiring medical re-examination or payment of extra premium; and Company "C" for 18 years, comparatively few such policies having been issued in the early years.

In Company "A" the percentage of disability claims from tuberculosis is much greater than in the other companies. This may be due to its liberality in granting such claims. The marked differences in the percentages from paralysis and locomotor ataxia are partly due to the method of selection, partly to the age of

the disability experience, and partly to the practice of Company "B," which made its disability benefits retroactive to all policyholders without payment of premium. The high percentage from blindness in Company "B" was chiefly due to the last-mentioned cause.

Causes of Disability.	Co. "A" Per Cent.	Co. "B" Per Cent.	Co. "C" Per Cent.
Tuberculosis	45	31	30
Insanity and paresis	23	23	31
Paralysis	6	14	7
Accident	4	3	4
Cancer and tumor	4	2	4
Heart disease	3	1	2
Blindness	2	5	1
Rheumatism	2	2	—
Locomotor ataxia	1	6	4
Bright's disease	1	½	1
Other causes	9	12½	16

An analysis of the disability claims due to tuberculosis in Company "A" showed that about three fourths of the insured in this group were under the average weight at date of application for insurance, but this is to be expected as the death rate from tuberculosis among light-weight persons is much higher than among those who are overweight. The large percentage of claims from tuberculosis may not be as serious as first appears because the average period over which the disability continues is short. In Company "A" within two and a half years after the claim was approved three fourths of the disabled had either died or recovered from the tuberculosis.

It was stated in the paper referred to that the proportions by cause of approved claims for disability would be affected by a number of conditions which would vary in different companies. These conditions are:

1. The length of the period over which the company has granted the benefits. Thus, a company which has been issuing disability benefits over a long period would have a proportionately larger percentage of claims on account of diseases of middle life, such as cancer.

2. The age after which disability claims are not allowed. Thus, a company granting disability benefits, which ceased at age 60 if the insured were not then disabled, would have a smaller percentage of diseases due to the breaking down of the human

organism than a company which had no limiting age for its disability benefits.

3. The rules of the company in limiting the disability benefits to certain types, or in granting such benefits freely. For example, a company accepting risks with a history of syphilis would have a large percentage of claims due to paresis and locomotor ataxia.

4. The degree of liberality governing the company's decision in admitting claims. For example, some companies will approve a claim on account of tuberculosis as soon as the insured has given up work as a result of the disease, while others will not admit the claim until it appears that the insured is not likely to recover.

5. The custom with regard to death losses under which the insured might properly have made, but did not make, a claim for disability benefits. Some companies will grant the benefit only if claim has been made, while other companies make an investigation in connection with all deaths due to insanity and if it appears that the insured could have made a claim for the benefit but did not, then an application for the benefit is considered to have been made.

6. The form of policy, which may result in claims being deferred until immediately before the anniversary of the policy. When the policy provides that the first instalment shall be paid one year after the anniversary following the date of application for the benefit there is no incentive to file claim until a short time prior to the anniversary of the policy succeeding the commencement of disability.

7. The practice of granting or not granting disabililty benefits on policies in force before disability benefits were introduced. A company automatically extending the waiver of premium benefit to all policyholders under the age of 60 without additional charge would have some cases of disability on account of lives which would not be acceptable under the new provisions.

8. The character of the benefits allowed on total and permanent disability. A company limiting the benefit to waiver of premium would have different proportions of claims on account of the various causes from those experienced by another company granting the annuity feature—the element of fraud in the former case would probably be less than in the latter.

Recovery from Disability.

Claims for disability benefits are generally approved upon the receipt of due proof that the insured is totally and will *presumably* be permanently disabled. This means in practice that a number of claims are approved under which the disability later proves to be only temporary, and the more liberal the company in granting disability benefits, the greater will be the percentage of recoveries. As an illustration it may be mentioned that in the case of Company "A," the insured had recovered under 26 per cent. of the claims which had been approved one year or more prior to the date of the investigation. (Vol. XIX, p. 225.)

There are instances of recurrence of disability after recovery. In Company "A" the cases of recurrence of disability after recovery were 8 per cent. of the total recoveries. These were all cases in which the insured had recovered a sufficient length of time before recurrence of the disability to have paid a premium to the company.

Selection of Risks for Disability Benefits.

The selection of risks to which disability benefits may be granted does not run parallel with the selection of risks to which life insurance may be given. The larger the disability benefit the greater must be the care exercised in eliminating applicants who are very likely to incur accidents causing disability and also those who through heredity, environment or occupation are liable to contract diseases which may cause permanent and total disability. Taking in the first place occupations, it may be said for example that the risk of loss of limbs is so much greater among locomotive firemen and yardmen in railroads that a premium much more than the regular premium for disability benefits would be required to cover the hazard. To advance the age a few years would not cover the increased hazard of disability, although it would cover the hazard of death. Two courses are open,—

1. (*a*) To determine the hazard of death and charge a suitable premium therefor, and
   (*b*) to fix the hazard of disability in relation to the normal and charge the proper extra premium.
2. To decline to grant disability benefits in all occupations which involve a high risk of disability.

It would not be feasible to give a list of such occupations, because a company granting the waiver of premium benefit might accept one group of risks while another company would decline to grant to the same group the disability benefits payable during lifetime with the face of the policy payable at death. A few occupations, however, may be given in which the practice of the companies is fairly uniform. Disability benefits are usually not granted to soldiers, sailors, caisson workers, dynamo tenders, employees of gunpowder factories, structural iron workers, miners, quarrymen and many connected with railroad service, such as yardmasters, signal maintainers, engine firemen and yard switchmen. There are other occupations in which the extra hazard of disability is not from accident but from disease. Men liable to tuberculosis from such trades, for example, as stone cutting and glass blowing may be taken as types.

Experience has taught companies that there are certain medical impairments which should exclude applicants from receiving disability benefits unless an extra premium is charged. Owing to the large percentage of disability claims from tuberculosis the companies have generally been chary of granting such benefits to young persons who are under the average weight and have a family history of tuberculosis. The number of disability claims from paresis, locomotor ataxia and paralysis has resulted in most of the companies declining to grant disability benefits to persons with a history of syphilis, even though it may be considered as cured. Persons who have a family history of two cases of insanity or epilepsy are refused disability benefits by some companies. A personal history of epilepsy or insanity is generally a bar to disability benefits. Where a man has lost a leg, an arm or an eye, disability benefits are rarely granted, although occasionally a policy may be issued with a clause providing that if the disability is the result of the impairment the disability benefits shall then become null and void.

Granting of Disability Benefits to Women.

When disability benefits were first granted by insurance companies women were considered ineligible for that benefit, although they were granted life insurance. The practice soon developed, however, of granting such benefits to women who were self-supporting, whether spinsters, widows or divorcees. At a later

date a few companies included women who were not earning a living but who had an income of their own from investment. The practice is now broadening with regard to married women, so that several companies are granting disability benefits to married women who have a business of their own or who spend a considerable part of the day in their husbands' stores, or who have a separate income of their own. In general, however, married women are not granted disability benefits, partly because of the difficulty of maintaining on the part of the company that they are capable of "following a gainful occupation" and partly on account of the possible incentive to fraud.

PROVISIONS REGARDING SURRENDER VALUES AND DIVIDENDS.

Where the disability benefits cover only the waiver of future premiums the same loan and cash values are allowed after the insured has become disabled as if he had continued to pay premiums. The paid-up and extended insurance features are not affected by the clause covering waiver of future premiums, as either might be of great value to the insured in event of his subsequent recovery after having had one or more premiums waived.

Where the disability benefit consists of the payment of the face amount in instalments, it is not practicable to allow the surrender values to remain unchanged after payment of the instalments has commenced as the original loan or cash value may eventually exceed the outstanding balance of the sum insured. If, for example, one tenth of the face of the policy is payable during each year of disability for a period of ten years, the insured could borrow the full loan immediately before the last disability payment was due, and such loan might be greater than the last disability payment. It is a common practice therefore to provide that the cash, loan, and paid-up values are subject to reduction in the proportion which the amount paid under the disability benefit bears to the face of the policy.

Where an annuity is payable so long as the insured is disabled and the face of the policy is due at death, the policy is in the same position on recovery from disability as if the premiums had been regularly paid, and accordingly the surrender values are not altered.

No additional cash or other surrender value is allowed on account of the disability benefit, policies generally providing that such

values will be based on the *regular* reserve under the American or the Actuaries' Table.

In some participating policies it is stated that the dividends shall be the same after premiums are waived as upon corresponding policies under which the premiums are being paid. In other participating policies no mention is made of the effect on dividends of waiving the premium, and in that event, the treatment of dividends would depend on the methods adopted by the company. If the calculation of the net premium for disability has been made to cover the waiving of the gross premium under the policy, then the company can afford to pay the same dividend during disability as under corresponding policies on which the regular premiums are being received. If, on the other hand, the calculations of the extra premium for the disability benefit have been made on the basis of the net premium for the policy, there would be no dividend from loading, but some companies pay a dividend from the interest in excess of the requirements on the reserve, less a charge for expenses of management. Whether any dividends should be granted from mortality savings or from profit on terminated policies on "active" lives would depend on the principles underlying the method of distributing the surplus of the company. When the disability benefit consists of instalments of the face of the policy or of a life annuity, some companies cease to pay dividends and others continue to pay either regular dividends on the unpaid portion of the policy in the case of the former or on the face of the policy in the case of the latter, while others pay excess interest earnings on the amount of the reserve. The method of calculating premiums, as already intimated, governs in this matter.

Legal Interpretation of Total Disability.

While disability benefits have been granted by fraternal orders for a quarter of a century, few cases have been taken to the courts for adjudication, and most of these cover points which are not of importance at this time to life insurance companies. The small number of cases which have been in legal controversy is due partly to the nature of the organization of the fraternal orders, and partly to a probationary period of six months before any payment is made under approved claims. During that probationary period these orders through their membership have excellent means of

keeping the applicant under careful observation. Quotations are of interest in two cases, the first being Hutcheson vs. Supreme Tent K.O.T.M. The Court made the following statement:

"The loss of a hand by a lawyer might interfere but slightly in the transaction of his business or in the performance of his work, while to a man who has learned a particular trade by which he had always earned his living and was entirely ignorant of all other trades or business, it might prove to be a much more serious disability."

The foregoing indicates the difficulty of defining in policies what constitutes total and permanent disability, especially among capitalists, those who have retired from business, and married women who are engaged in household duties.

In the case of McMahon vs. Supreme Council 54, Mo. App. 468, where the policy granted benefits in event of the insured becoming "totally and permanently disabled from following *his* usual occupation," it was held that "the total disability would occur where the party was prevented from following an occupation whereby he could obtain a livelihood, and that in determining whether such a disability exists in a given case, both the mental and physical *capabilities* of the insured should be considered."

The majority of cases of this nature before the courts are against accident and casualty companies where a weekly payment or a lump sum is due the insured on his becoming totally disabled, but not necessarily permanently disabled. One of these cases is interesting (Neafle vs. Manufacturing Accident Indemnity Company), as the decision of the court permits a physician to do a certain amount of work and still be considered totally incapacitated. The court made the following statement:

"Total disability must of necessity to the case be a relative matter, and must depend largely upon the occupation and employment in which the party insured is engaged. One can readily understand how a person who labors with his hands would be totally disabled only when he cannot labor at all, but the same rule would not apply to the case of a professional man whose duties required the activity of the brain and which is necessarily impaired by serious physical injury. If a person engaged in the general practice of medicine and surgery is unable to go to his business and to his office and make calls upon any of his patients, but is confined to his bed as in this instance, and enabled only to exercise his mind on occasional applications to him for advice, he may be said to be totally disabled within the meaning of the provisions of this policy."

In his book on Insurance, Joseph A. Joyce in Section 3031

defines total disability within the meaning of an indemnity policy as follows:

"Total disability does not mean absolute physical disability on the part of the insured to transact any kind of business pertaining to his occupation. Total disability exists although the insured is able to perform occasional acts, if he is unable to do any substantial portion of the work connected with his occupation. It is sufficient to prove that the injury wholly disabled him from the doing of all the substantial and material acts necessary to be done in the prosecution of his business, or if his injuries were of such a nature and degree that common care and prudence required him to desist from his labor so long as was reasonably necessary to effect a speedy cure."

When there is a weekly indemnity payable during total disability, the question is different from that in which an annual premium is waived upon total and permanent disability. Under the former benefit, which is generally granted by accident and casualty companies, the claims are in the majority of cases for total disability of a temporary nature, while in the latter the claims of a temporary nature are eliminated, and, as a result, the outlay for the disability feature in life insurance policies is generally small compared with the death losses.

It is probable that very few disability claims will be referred to the courts for adjudication by life insurance companies, because the nature of the contracts permits the ultimate determination of the character of the impairment, the benefits being spread over several years. There has apparently been only one case against a life insurance company in connection with permanent and total disability benefits, namely, Buckner vs. Jefferson Standard Life Insurance Company (page 897 of 90 S. E.). A locomotive engineer who had lost one hand was not considered by the court as being permanently and wholly prevented from pursuing any and all gainful occupations.

COMBINED MORTALITY AND DISABILITY TABLES.

Sources of Statistical Information.

For the construction of disability tables it is essential to have material from which the rates of disability and the death rates among disabled lives may be determined.

In the United States and Canada the principal sources of such statistical information have been the records of the fraternal orders, the disability experience of the regular insurance companies being too recent to be of value. There are abundant

German statistics, but these are not considered applicable to conditions in the United States and Canada. For example, there have been published three disability experiences on Saxon, Prussian and Austrian miners, and three experiences covering different years of exposure on German railway employees. The investigation into Railway Employees by Zimmerman was divided into three sections: (*a*) Trainmen; (*b*) Office Employees; (*c*) Other than Trainmen and Office Employees. The statistics covering the office employees of German railroads are more nearly applicable to insured lives in companies in this country than groups (*a*) and (*c*), but the consensus of opinion among actuaries is that even these statistics are not suitable. The German government has published statistics relating to compulsory insurance against permanent invalidity, but as these apply to working men who were not medically examined for the benefits, the rates of disability and of mortality would not be the same as among the insured in companies transacting "ordinary" business in the United States and Canada, whose membership is drawn from a different social class. Summaries of the German statistics appear in the papers of S. H. Pipe, F. B. Mead, and C. W. Jackson.

Léon Marie has published a table based upon the material collected from the six largest railroad companies in France, eliminating from the number disabled those who, after consultation with physicians, did not appear to be permanently incapacitated. These tables also are not applicable to disability insurance in this country.

No statistics have been published in Britain specifically covering total and permanent disability. C. W. Jackson (T. A. S. A., X, 490) however, used the sickness experience of the Manchester Unity of Great Britain, assuming that those who were sick for more than two years were permanently and totally disabled.

In the paper by S. H. Pipe (T. A. S. A., XI, 172) there appear rates of disability and of death among disabled lives based on the experience in the Province of Ontario of the Independent Order of Foresters. The statistics cover the years 1887 to 1907 inclusive. The years of exposure amount to 316,364, and the cases of disability to 467.

The statistics on which F. B. Mead (T. A. S. A., XI., 304) based his tables consisted of the experience among the Knights of the Maccabees from 1883 to October 1, 1909. There were 2,927 cases of disability.

In the tables prepared by Arthur Hunter (T. A. S. A., XII, 44) the statistics of three fraternal orders were used in which there were records of 2,627 disabled lives.

In a discussion on S. H. Pipe's paper, C. W. Jackson (T. A. S. A., XI, 401) gave data regarding the rate of disability among women. His statistics were based on the experience of a fraternal order called the "Ladies of the Maccabees," which was organized in 1892 and had in 1911 a membership of 120,000. He showed that the rate of disability among women was lower than in the experience among men as compiled by S. H. Pipe, and that the death rate among the disabled was also lower. A light rate of disability and a high death rate among the disabled results in a low net premium for the general types of disability benefits provided by life insurance companies. Under the same circumstances a light rate of disability and a low death rate among the disabled may result in practically the same net premium as a high rate of disability and a heavy death rate among disabled lives. It should not be assumed that life insurance companies would have as low a rate of disability among women as the fraternal orders. The latter have special facilities for detecting frauds.

Form of Table—"Active" and "Disabled" Lives.

To determine premiums and reserves for Disability Benefits it is necessary to have a Combined Mortality and Disability Table. Such a table can be readily constructed if we know for each age required:

(*a*) The rate at which the active lives become disabled;
(*b*) The rate of mortality among the disabled lives;
(*c*) The rate of mortality among the active lives, i. e., the lives that are not included among the disabled.

Three methods of obtaining a table to represent the mortality among active lives have been used in practice:

1. A table representing the mortality among active lives may be obtained by an investigation of these lives.
2. A published table of mortality may be adopted to represent the mortality among active lives:
3. A published table of mortality may be adopted to represent the total mortality among active and disabled lives. This

was the method used by Hamza in his paper presented to the Third International Congress of Actuaries. It is known as the Hamza process and has gained considerable prominence in this country.

Method No. 2 is only permissible when the published table adopted represents within reasonable limits what the mortality among active lives is known or believed to be. The student should note that if a table approximately represents the mortality among the whole body of policy-holders, it would not properly represent the mortality among the active lives because the active lives have a lower mortality than the disabled lives.

Pipe, Mead and Hunter in constructing their combined tables of mortality and disability used method No. 3 for obtaining the rate of mortality among active lives. They employed the American Table of Mortality to represent the total mortality. Throughout this study it will be assumed that the American Table of Mortality is employed to represent the total mortality of active and disabled lives. For convenience we shall refer to a table adopted for this purpose as a total mortality table.

The rate of disability adopted will usually be ultimate, i. e., it will be aggregate excluding the first few years of insurance.

Owing to the peculiar incidence of the mortality among disabled lives, it is customary to use a table aggregate in form but producing annuity values approximating those produced by select rates of mortality among disabled lives.

A section of a combined mortality and disability table is shown below:

COMBINED MORTALITY AND DISABILITY TABLE.

Age.	(1). r_x	(2). q_x^{i}	(3). q_x^{aa}	(4). l_x^{aa}	(5). d_x^{aa}	(6). t_x	(7). l_x^{ii}	(8). d_x^{ii}	(9). l_x	(10). d_x
15	.000587	.267	.00755	96,285	727	57	0	8	96,285	735
16	.000584	.254	.00747	95,501	713	56	49	19	95,550	732
17	.000581	.241	.00740	94,732	701	55	86	28	94,818	729
18	.000578	.229	.00740	93,976	695	54	113	32	94,089	727
19	.000575	.217	.00740	93,227	690	54	135	35	93,362	725
20	.000574	.205	.00742	92,483	686	53	154	37	92,637	723
21	.000572	.193	.00746	91,744	684	52	170	38	91,914	722
22	.000571	.182	.00750	91,008	683	52	184	38	91,192	721
23	.000570	.171	.00755	90,273	682	51	198	38	90,471	720
24	.000570	.161	.00761	89,540	681	51	211	38	89,751	719

l_x^{aa} = the number of active lives at age "x,"
l_x^{ii} = the number of disabled lives at age "x,"
d_x^{aa} = the number dying while active between ages "x" and "$x + 1$,"
d_x^{ii} = the number dying, while disabled, between ages "x" and "$x + 1$,"
i_x = the number of active lives becoming disabled between ages "x" and "$x + 1$,"
r_x = the probability of an active life aged "x" becoming disabled within a year,
q_x^i = the probability of a disabled life aged "x" dying within a year,
q_x^{aa} = the probability of an active life aged "x" dying within a year while still active,
l_x = the number living at age "x" according to the total mortality table. This consists of the number of active living and the number of disabled living,
d_x = the number dying between ages "x" and "$x + 1$" according to the total mortality table. This consists of active lives dying and disabled lives dying between ages "x" and "$x + 1$."

It will be observed that the number of persons living at any age according to the total mortality table is divided into active and disabled lives, that the sum of the number of active and the number of disabled lives is the number living according to the total mortality table, and that the sum of the deaths among active lives and of the deaths among disabled lives is the number of deaths according to the total mortality table. The assumption is usually made that at age 15 all the lives in the total table are active lives.

The student must keep in mind that in the division of the mixed lives into active lives and disabled lives the combined mortality of these two divisions must remain the same as in the total table.

If the Hamza process of determining the mortality among active lives is adopted it is not necessary to investigate the mortality among both active and disabled lives. It is necessary to investigate only the mortality among the disabled lives, thus obtaining the deaths among disabled lives—the deaths among

active lives being obtained by subtracting the deaths among the disabled lives from the total deaths according to the mortality table adopted to represent the mortality among active and disabled lives.

Rate of Disability.

It is evident that the values of r_x derived from the experience of a company or fraternal order will depend to some extent on the rates of mortality among the active lives, for the greater this mortality the smaller the number exposed to risk of disability. There is, therefore, a theoretical objection to using the values of r_x obtained in this way in conjunction with the total mortality table, and the rate of disability derived should be one which is independent of the rate of mortality. Such a function is found in the "absolute" rate of disability, r'_x, which may be defined as the annual rate of disability among active lives who do not die during the year; i.e. it is the rate per unit exposed for the full year whereas r_x is the rate per unit entering on the year.

To investigate the relation between r'_x and r_x; let E''_x be the number exposed to risk of either disability or death in the year from age x to age $x + 1$, so that in determining E''_x active lives passing out of observation through death or disability are taken as exposed for the full year. And let E'_x represent the corresponding number exposed only to risk of disability. Then in calculating E'_x the deaths as well as all other terminations, except those due to disability, must be taken as passing out of observation at the nearest duration, so that, on the assumption of a uniform distribution of the deaths among active lives,

$$E'_x = E''_x - \tfrac{1}{2}d^{aa}_x$$

while

$$q^{aa}_x = \frac{d^{aa}_x}{E''_x} \tag{1}$$

$$r_x = \frac{i_x}{E''_x} \tag{2}$$

$$r'_x = \frac{i_x}{E'_x} = \frac{i_x}{E''_x - \tfrac{1}{2}d^{aa}_x} = \frac{i_x}{E''_x(1 - \tfrac{1}{2}q^{aa}_x)}. \tag{3}$$

From (2) and (3) we have

$$r_x = r'_x(1 - \tfrac{1}{2}q^{aa}_x) \tag{4}$$

as the relation sought; while in the combined mortality and disability table we have the analogous relations

$$r_x = \frac{i_x}{l_x^{aa}} \tag{5}$$

$$r'_{x\frac{1}{2}} = \frac{i_x}{l_x^{aa} - \frac{1}{2}d_x^{aa}} \tag{6}$$

From equation (4) it is evident that the greater the mortality among the active lives the less will be the rate of disability r_x although the absolute rate of disability r'_x remains unaltered. To be theoretically strictly correct r'_x as ascertained from the disability data should be combined with the rates of mortality adopted for active lives and the other functions including r_x should be determined from the relations existing among these functions. The slight error involved in using the values of r_x that have been obtained directly from the disability data is, however, frequently ignored in practice.

The first step in deriving the rate of disability from the experience of an insurance company or fraternal order is to determine what shall be considered disability, so that a definite line may be drawn between deaths among active lives and deaths among disabled lives. As nearly all deaths are preceded by a period of a few days or a few weeks of total disability the problem is not an easy one. Furthermore, the rule that is observed in drawing the line between deaths among active lives and deaths among disabled lives will affect the resulting rates of disability and the death rates among disabled lives. If, for example, all lives that have been disabled 30 days or more be considered as disabled, and subsequent deaths among them be considered as deaths among disabled lives, the resulting rates of disability and the death rates among disabled lives will be very much different from those that would be obtained by considering as disabled only those that have been disabled 60 days or more. It is sometimes provided in policies containing disability benefits that disability must be in existence 60 days before claim for disability benefits will be allowed.

It is moreover imperative that the terms of the policy contract and the practice of the Company in allowing claims be carefully considered as to possible effect upon the number of disability

claims. Where the benefit is entered into on the anniversary next succeeding the receipt of proof of disability many claims will not be filed until shortly before the anniversary and, moreover, the company may, in order to eliminate useless investigations, purposely advise deferring the filing of proof of disability until shortly before the anniversary—thus causing many deaths among lives that have been disabled less than one year to be regarded as deaths among active lives. If an investigation of the practice of a company or fraternal society shows that the latter is the case, then it will not be practicable to obtain directly from the experience either r_x or r'_x; but it would be possible to ascertain p_x^{ai}, the probability of an active life aged "x" becoming disabled during the year and surviving to the end of the year.

When the benefit dates from the receipt of proof of disability and becomes available six months or one year thereafter, it is necessary to carefully consider the practice of the company in recording deaths occurring between the time the proofs are filed and the time the benefit becomes available. In this case the policyholder may be depended upon to promptly file proof of disability, so that the company's records may be in such a form that r_x or r'_x may be derived from them. But, on the other hand, the company may not complete its investigation of the case until shortly before the benefit becomes available and may regard all deaths occurring in the interim as deaths among active lives. In the latter case the records could be used to determine the probability of becoming disabled and surviving six months or one year thereafter, but could not be used to determine r_x or r'_x.

When the benefit is entered into immediately upon receipt of proof that the policyholder has been disabled for 30 or 60 days, then r_x or r'_x may be ascertained from the experience. In this case all deaths occurring within a 30- or 60-day period following disability would be recorded as deaths among active lives and only those lives that survived the period of 30 or 60 days would be recorded as disabled. In view of the very heavy death rate in the year following the occurrence of disability especially in the first few months of that year, it is obvious that the resulting rates will be greatly affected by the length of the probationary period. If all deaths occurring after disability has been in existence for thirty days be regarded as deaths among disabled lives, then the rate of disability and also the

rate of death among disabled lives will be very much higher than if the probationary period be 60 days.

In investigating the experience of the company or fraternal society, it is, therefore, necessary to have clearly in mind what probability can be derived from the records, so that the resulting probability may be correctly combined with the total mortality table for the purpose of calculating premiums and reserves. From what has been said the records of the company or fraternal society may generally be used to derive one of the following:

(*a*) r_x, the probability of an active life aged "x" becoming disabled within a year;

(*b*) r'_x, the absolute annual rate of disability among active lives aged "x" at the beginning of the year;

(*c*) $p_x^{a'}$, the probability of an active life aged "x" becoming disabled and surviving to the end of the year.

The investigation should be made in the select form, so that the effect of selection may be measured and so that the experience of the early policy years may be excluded in forming the ultimate table. The policy year method should be used.

The following symbols will be used in determining the rate of disability:

$n_{[x]}$ = new entrants at nearest age "x."

$s^{aa}_{[x]+t}$ = active survivors at age "$[x]+t$," that is, those who entered at age "x" and who have been "t" years on the books at the anniversary in the year the observation began and who are still active. It should be noted that $s^{aa}_{[x]+0}$ is taken as zero in the following formulæ, *as it is the same as* $n_{[x]}$ which is separately tabulated.

$i_{[x]+t}$ = lives who entered at age "x" and who became disabled between the ages "$[x]+t$" and "$[x]+t+1$."

$d^{aa}_{[x]+t}$ = deaths among active lives who entered at age "x" occurring between ages "$[x]+t$" and "$[x]+t+1$."

$w^{aa}_{[x]+t}$ = withdrawals among active lives who enter at age "x" and with nearest duration of "t" years.

$e^{aa}_{[x]+t}$ = active lives existing at age "$[x]+t$" at policy anniversary in the last year of observation.

$r_{[x]+t}$ = the probability of an active life aged "$[x]+t$" becoming disabled within a year.

$r'_{[x]+t}$ = the absolute annual rate of disability among active lives in the $(t+1)$th year of duration who entered at age "x."

$E'_{[x]+t}$ = exposed to risk of disability in the $(t+1)$th year of insurance.

$E''_{[x]+t}$ = the number of active lives exposed to risk of death or disability in the $(t+1)$th year of insurance.

(*a*) *To determine* $r_{[x]+t}$:

$$(7)\quad E''_{[x]+t} = n_{[x]} + \sum_0^t (s^{aa} - w^{aa} - e^{aa})_{[x]+t} - \sum_0^{t-1} (d^{aa} + i)_{[x]+t};$$

$$(8)\quad r_{[x]+t} = \frac{i_{[x]+t}}{E''_{[x]+t}}.$$

(*b*) *To determine* $r'_{[x]+t}$

on the assumption that the deaths among active lives are uniformly spread over the year i. e. that the number of deaths among active lives during the first half of the $(t+1)$th policy year among lives who entered at age x is $\frac{1}{2} d^{aa}_{[x]+t}$;

$$(9)\quad E'_{[x]+t} = n_{[x]} + \sum_0^t (s^{aa} - w^{aa} - e^{aa})_{[x]+t} - \sum_0^{t-1} (d^{aa} + i)_{[x]+t} - \tfrac{1}{2} d^{aa}_{[x]+t};$$

$$(10)\quad r'_{[x]+t} = \frac{i_{[x]+t}}{E'_{[x]+t}}.$$

(*c*) *To determine* $p^{at}_{[x]+t}$:

$$(11)\quad p^{at}_{[x]+t} = \frac{j_{[x]+t}}{E''_{[x]+t}},$$

where $j_{[x]+t}$ represents the lives who entered at age x, and who became disabled during the $(t+1)$th policy year *and survived to the end of the year.*

Whichever probability is obtained, $r_{[x]+t}$, $r'_{[x]+t}$, or $p^{at}_{[x]+t}$, the separation of the data into select and ultimate experience should be made in the same manner as when investigating the rate of mortality.

Existing Tables of Rates of Disability.

Until recently, the only reliable tables of disability in existence were those compiled in Germany, the one most frequently quoted being Zimmerman's, on the employees of German railroads. A

synopsis of these statistics appears in J. I. A., XXIX, 306, but is not quoted in this study because they are not of direct value to life insurance companies, as the lives were not medically examined. They would not be reliable guides to the disability rate even among railroad men in the United States, because the hazard from occupation differs in the two countries.

No life insurance company in America or Canada has published its disability experience in detail because none has had an extended experience of sufficient magnitude to justify basing premiums or reserves thereon. The actuarial profession is principally indebted for its information on the subject to the fraternal orders which have placed their data at the disposal of several actuaries. The members in these orders were examined in the same way as the insured in life insurance companies. The disability benefit in these orders generally consists of payment of the sum insured in instalments.

The first of the disability tables based upon the experience of an American fraternal society appeared in 1909 in a paper prepared by Pipe (T. A. S. A., XI, 175), and consisted of the experience in the Province of Ontario of a large Canadian fraternal society. The experience of that society is given by him (T. A. S. A., XII, 334). The rates of disability which he gave in this discussion were obtained by excluding the first four years of insurance, and are on an ultimate basis.

The rates of disability appearing in Mead's papers are based on the experience of an American fraternal order and first appeared in a book by Abb Landis, entitled "Analysis of Fraternal Societies" (p. 150). Mead modified the rates at the higher ages. The experience is an aggregate one.

The rates given by Hunter were based on the experience of the two orders already mentioned, with the data of a third fraternal society. An ultimate rate was derived from the statistics, the experience of the first five policy years being excluded.

In studying the following table of comparison the different methods of construction should be kept in mind.

Considering the comparatively small amount of data available at the present time, the ultimate rates of Pipe and Hunter are as close as could be expected, the principal divergences being at the youngest and oldest ages. Mead's high rate at age 65 is due to the assumption that all lives would be disabled at age 80 and to an adjustment from about age 60 to that age.

RATES OF DISABILITY PER 1,000 BASED ON FRATERNAL SOCIETIES' EXPERIENCE.

Age.	Pipe (Ultimate).	Mead (Aggregate).	Hunter (Ultimate).
20	.37	.12	.515
25	.51	.25	.528
30	.61	.37	.561
35	.66	.48	.642
40	.76	.69	.832
45	1.03	.90	1.151
50	1.74	1.24	1.696
55	2.74	2.55	2.752
60	5.00	8.30	5.402
65	10.80	28.02	12.388

The extent to which an aggregate rate departs from the ultimate rate depends on the effect of medical selection in reducing the disability rate in the early policy years. The greater the effect of such selection and the shorter the period covered by the investigation, the greater will be the difference between the aggregate and ultimate rates of disability. A table was given by Hunter showing the effect of selection for five years, the figures indicating that he could have prepared his ultimate rate by eliminating the first four, instead of the first five policy years. A column has been added to the following table showing the results of applying Pipe's ultimate rate to the exposed to risk in Hunter's experience in the first four policy years.

EFFECT OF MEDICAL SELECTION.

Hunter's Fraternal Experience.

Policy Year.	Expected Claims by Hunter's Ultimate Rates of Disability.	Actual Number Becoming Disabled.	Ratio of Actual to Expected.	Ratio of Actual to Expected by Pipe's Ultimate Rates of Disability.
1st	483	46	10%	12%
2d	386	201	52	65
3d	330	231	70	79
4th	288	222	77	76
5th	254	265	104	

It appears from the foregoing that the effect of medical selection is strong but of short duration.

MORTALITY AMONG DISABLED LIVES.

In practice there are different customs governing the approval of claims and the payment of benefits. In fraternal orders there

is generally a probationary period of six months required after the disability occurs, but the method of procedure in different orders varies. For instance, a Canadian order requires a probationary period of six months after notice of claim before any payment is made; while an American order requires proof on filing the claim that the disability has already existed for six months. Consequently, assuming that claim was made when disability began, an actuary in dealing with the data of the former would have the disabled lives under observation from the beginning of disability, and under the latter not until six months after disability commenced, any deaths occurring in the preceding six months being considered as among the active lives. It follows that in comparing rates of disability the treatment during the probationary period should be taken into account. When the experience of life insurance companies is tabulated, the rules regarding the treatment of claims for disability should be known. Where the sum insured is payable as an annuity commencing six months or a year after disablement, it is the custom to make an examination of the insured as soon as the claim for disability is made; while in companies in which the disability benefit consists of waiving the premiums, an investigation into the physical condition of the insured is not made until shortly before the premium is due. In the former case there would be included as deaths among the disabled, some who die before the next premium is due, while in the latter case such deaths would appear as deaths among active lives. Accordingly, the companies with such an annuity benefit would ordinarily show a higher rate of disability and a higher rate of mortality among the disabled than those with waiver of premium only. As there are different methods of determining the date on which the disability annuity commences, and as several companies have changed their practice in this regard, care must be taken to note the interval between the beginning of the disability and the approval of the claim. The same company, for example, might have at one time agreed in its policy to grant the first instalment or the first annuity one year after approval of the claim; while at another period its policy might have provided for such instalment being paid one year after the anniversary of the policy following the approval of the claim.

The methods of constructing mortality tables of the disabled are

the same as those used in constructing the ordinary mortality tables on insured lives. The date of entry is the date at which the insured becomes disabled, or the date at which the insured is definitely regarded as being no longer an active life. Owing to the fact that the mortality curves of disabled lives are entirely different from those of other insured lives, select or analyzed tables are generally preferred, although an aggregate table may be used for the sake of simplicity in the calculation of net premiums and reserves. This difference may be seen from the German Government Table of 1906, based on the experience during 1891 to 1903, of over 440,000 annuities granted to invalid German workmen, of which over 200,000 had terminated by death at the close of observation:—

DEATH RATE PER 1,000 AMONG DISABLED WORKMEN.
GERMAN GOVERNMENT EXPERIENCE OF 1906.

Age at Date of Disability.	Disability Years.								
	1	2	3	4	5	6	8	10	11 and Over.
20	600	350	213	147	106	80	54	41	37
30	479	278	171	120	91	74	54	44	40
40	370	215	135	100	82	70	55	48	48
50	266	156	105	86	76	69	59	59	63
60	172	111	90	81	76	75	83	92	101
70	131	108	107	112	118	124	139	155	174

There are two things to be noted in connection with the foregoing table:

1. That the mortality generally decreases for a number of years after the insured has become disabled;

2. That the mortality in the 1st and 2d policy years after disability occurs decreases with advancing age, and in several subsequent years it decreases except at the old ages.

The high mortality among the younger men in the early years of disability is due partly to diseases like tuberculosis, which are more prevalent among the young than among the old, and which cause a heavy mortality following the occurrence of the disability.

The decrease in mortality with advancing policy years is due to two distinct types of impairment in the exposed. One type consists of those who are seriously sick from some disease, causing a high mortality; while the other type consists of those who have become blind or have had both legs amputated or some other affliction, not seriously decreasing their chances of

longevity. The former class would probably have a decreasing mortality, as the strongest of them would outlive their impairment; while the latter would probably have a mortality only slightly higher than that of the population. The combined mortality of these two types after several years following disablement would result in a mortality not greatly different from that among the population in general.

The following table shows the death rate in the first disability year as given in three tables based on data of the fraternal orders in the United States and Canada compared with the German experience on workmen:

DEATH RATE DURING FIRST YEAR AFTER DISABILITY AMONG DISABLED LIVES.

Age.	Mead.	Pipe.	Hunter.	German Workmen.
25	.399	.447	.400	.537
35	.347	.414	.327	.424
45	.306	.376	.290	.317
55	.262	.330	.248	.217
65	.195	.246	.190	.143

In the preparation of these rates of mortality Mead did not include the deaths occurring within six months after claim of disability, for the reason that notice and proof of disability were not filed until after disability had existed six months; Pipe included all deaths from the time the insured claimed to be disabled; while Hunter excluded all cases in which the death occurred in the same policy year in which the insured made claim for disability. The German workmen were probably exposed to a higher rate of accident and had not as favorable conditions for recovery from sickness as the members of American and Canadian fraternal orders.

The difference in the mortality of the first year after disablement derived by the three investigators of the experience of fraternal orders is mainly due to diverse treatment of disability claims by the different orders. By re-arranging the data so as to follow actual policy years and not years after disablement, Pipe showed that his death rate in the first year was nearer to Mead's than the foregoing table implied. Hunter's method of disregarding the deaths occurring in the policy year in which disablement occurred though strictly correct as applied to waiver of premiums when payable yearly, was not claimed to be an accurate

method, but one which would increase the net premiums, thereby adding a small factor of safety. If we assume that the average duration between disability and the following policy anniversary is six months, then Hunter's and Mead's tables may be regarded as starting from the same time, that is, six months after disability, and can be compared without having to make allowance for different methods of recording the data. A synopsis of these two tables is given in order that the incidence of mortality may be clearly seen.

DEATH RATE PER 1,000 AMONG DISABLED LIVES.

Age.	Table.	Year of Disability.							
		1	2	3	4	5	6	7	8
25	Hunter.....	400	183	122	76	58	49	40	34
	Mead.....	399	190	108	54	43	33	27	20
35	Hunter.....	327	145	97	64	48	43	39	35
	Mead.....	347	154	85	64	52	39	32	26
45	Hunter.....	290	140	112	83	59	52	47	45
	Mead.....	306	148	104	78	65	58	54	51
55	Hunter.....	248	147	120	100	83	78	76	75
	Mead.....	262	142	116	89	80	75	73	75

Both tables show similar characteristics, though there are marked divergences.

In the ordinary mortality table the death rate increases with advancing age and with time elapsed since selection; accordingly the value of an annuity at the earlier ages is less on a select than on an aggregate basis. The reverse is the case with disabled lives, the use of an aggregate mortality table for disabled lives resulting in annuity values which are very low at the younger ages as compared with select values, since the death rate for the first few years following disability is high. To produce accurate values of annuities, a select rate of mortality must be used, but, for the sake of simplicity, an annuity value based on the mortality table excluding the first year after disability has been used in practice.

In the preceding explanation no attention has been paid to the question of what adjustment would be necessary if allowance were to be made for the probability of recovery. No such allowance has been made in any calculations based on experiences in America, the material being too meagre to render possible any

reliable estimate. If a sufficient body of experience were available, there would be no serious difficulty in making the necessary adjustments for recoveries in so far as waiver of premium benefits and disability annuity were concerned. Recovery in that case would be treated as having the same effect on the value of an annuity on a disabled life as a death, and the case would be considered as an active life at the attained age. When instalment benefits are in question, this method would not apply, because upon recovery, the policy is not restored to the condition of one under which no disability benefit had been paid. The company in effect continues to lose interest and usually premiums on so much of the sum insured as has already been paid in instalments. In event of recovery after receipt of a first instalment, the second instalment would not fall due unless the insured again became disabled. Thus the benefit in respect of the second year of disability might mature a year later than that for the first year, or at any subsequent period covered by the terms of the insurance. It would be necessary, therefore, to find the value as at the normal date (one year after the payment of the first instalment) of the benefit in respect of the second year of disability for all possible years in which it might become payable, and so on for each other year of disability. Such a process would be extremely laborious, because it would have to be carried out for every age of commencing disability. As disability premiums are generally of small amount and as the relief afforded to the companies by recoveries can never be more than a moderate proportion of the total cost, the labor required to give effect to the possibility of recovery is not likely to be undertaken in practice, and for this reason detailed methods of investigation and formulæ for premiums and reserves are not given here.

Construction of Combined Mortality and Disability Table.

As has been previously remarked, it is not always practicable nor advisable, when investigating the experience of a life insurance company or fraternal society, to determine r_x directly. The functions obtained from such an investigation may be

(*a*) r_x, the probability of an active life aged "x" becoming disabled within a year, or

(*b*) r'_x, the absolute annual rate of disability among active lives aged "x" at the beginning of the year, or

(*c*) p_x^{ai}, the probability of an active life aged "x" becoming disabled and surviving to the end of the year.

Whichever of these probabilities is available, it must be combined with the probability of death among disabled lives and with the probabilities from the total mortality table, assuming the Hamza process to be used. The process of constructing the combined mortality and disability table depends upon what functions are available.

(*a*) *When r_x is given.*—The values of l_x and d_x are taken from the total mortality table, usually the American Experience Table of Mortality, and the values r_x and q_x^i are obtained by investigation. From the relations existing among these functions the other functions may be determined. From the following equation i_x may be obtained:

$$i_x = r_x l_x^{aa}. \tag{12}$$

The number of disabled lives living at age "x" comprises the survivors from the number of disabled lives living at age "$x - 1$" increased by the number of survivors from those becoming disabled between ages "$x - 1$" and "x." On the assumption of an even distribution of disability and of mortality throughout the year, we have the relation

$$d_x^{ii} = q_x^i(l_x^{ii} + \tfrac{1}{2}i_x), \tag{13}$$

$$l_{x+1}^{ii} = l_x^{ii} + i_x - d_x^{ii}. \tag{14}$$

As the sum of the number of active lives and the number of disabled lives is the number living according to the total mortality table and as the sum of the number of deaths among active lives and the number of deaths among disabled lives is the number of deaths according to the total mortality table, the following relations obtain,

$$l_x^{aa} = l_x - l_x^{ii}, \tag{15}$$

$$d_x^{aa} = d_x - d_x^{ii}. \tag{16}$$

It will be observed that decrements to the l^{aa} column are of two kinds, namely deaths among the active lives and lives becoming disabled. Translated into symbols

$$l_x^{aa} - d_x^{aa} - i_x = l_{x+1}^{aa}. \tag{17}$$

At the initial age of the table, usually age 15, $l_x^{aa} = l_x$ and $l_x^{ii} = 0$. By means of equations (12) and (13), i_x and d_x^{ii} may

be obtained for the initial age; and by means of equations (14) and (15), l_x^{ii} and l_x^{aa} for the next higher age are obtained. The process is repeated until the entire table is completed. The values of d_x^{aa} may then be obtained by means of equation (16). The only way the student will become thoroughly familiar with the method of constructing a table is to work out the various values for a few ages.

(*b*) *When r'_x is given.*—As in the previous case the values l_x and d_x are taken from the total mortality table and the values r'_x and q_x^i are derived from investigation. Making the assumption that deaths among active lives are evenly distributed throughout the year,

$$i_x = r'_x(l_x^{aa} - \tfrac{1}{2}d_x^{aa}) \tag{18}$$

and making the assumption that the number becoming disabled are evenly distributed throughout the year

$$(l_x^{ii} + \tfrac{1}{2}i_x)q_x^i = d_x^{ii}. \tag{19}$$

Substituting the value of i_x from equation (18) we obtain

$$[l_x^{ii} + \tfrac{1}{2}r'_x(l_x^{aa} - \tfrac{1}{2}d_x^{aa})]q_x^i = d_x^{ii}. \tag{20}$$

Remembering that $d_x^{ii} + d_x^{aa} = d_x$ and substituting the value of d_x^{ii} from equation (20) we obtain

$$\begin{gathered}
[l_x^{ii} + \tfrac{1}{2}r'_x(l_x^{aa} - \tfrac{1}{2}d_x^{aa})]q_x^i + d_x^{aa} = d_x, \\
q_x^i l_x^{ii} + \tfrac{1}{2}r'_x \cdot q_x^i l_x^{aa} - \tfrac{1}{4}r'_x q_x^i d_x^{aa} + d_x^{aa} = d_x, \\
d_x^{aa}(1 - \tfrac{1}{4}r'_x q_x^i) = d_x - q_x^i l_x^{ii} - \tfrac{1}{2}r'_x \cdot q_x^i \cdot l_x^{aa},
\end{gathered}$$

$$d_x^{aa} = \frac{d_x - q_x^i \cdot l_x^{ii} - \tfrac{1}{2}r'_x \cdot q_x^i \cdot l_x^{aa}}{1 - \tfrac{1}{4}r'_x \cdot q_x^i}. \tag{21}$$

At the commencement of the table, r'_x and q_x^i being known, l_x^{ii} being zero and l_x^{aa} being equal to l_x, d_x^{aa} may be obtained by means of formula (21). Substituting the value of d_x^{aa} so found in equation (18) i_x is obtained. We can get d_x^{ii} from the relation $d_x^{ii} = d_x - d_x^{aa}$.

As this process may appear to the student somewhat complicated, a convenient worksheet is presented. Values of $(1 - \tfrac{1}{4}r'_x \cdot q_x^i)$ and $\tfrac{1}{2}r'_x \cdot q_x^i$ are first calculated for each age and are inserted in the table. Values of r'_x, q_x^i, l_x, and d_x should also be inserted for each age before commencing to calculate the other functions.

Column (1) = age,
" (2) = d_x,
" (3) = q_x^i,
" (4) = (3) × (14) = $q_x^i \cdot l_x^{ii}$,
" (5) = $\frac{1}{2} r_x' \cdot q_x^i$,
" (6) = (5) × (10) = $\frac{1}{2} r_x' \cdot q_x^i \cdot l_x^{aa}$,
" (7) = (2) − (4) − (6),
" (8) = $(1 - \frac{1}{4} r_x' \cdot q_x^i)$,
" (9) = (7) ÷ (8) = d_x^{aa},
" (10) = $l_x^{aa} = l_{x-1}^{aa} - d_{x-1}^{aa} - i_{x-1}$,
" (11) = r_x',
" (12) = (11) × [(10) − ½(9)] = i_x,
" (13) = l_x,
" (14) = (13) − (10) = l_x^{ii},
" (15) = (2) − (9) = d_x^{ii},
" (16) = (12) ÷ (10) = r_x.

(*c*) *When p_x^{ai} is given.*—At the initial age of the table $l_x^{aa} = l_x$ and $l_x^{ii} = 0$. As p_x^{ai} is the probability of a life aged "x" becoming disabled and surviving to the end of the year, at the initial age, say 15,

$$p_{15}^{ai} \cdot l_{15}^{aa} = l_{16}^{ii},$$

then as $l_{16}^{aa} = l_{16} - l_{16}^{ii}$, the value for the next higher age l_x^{aa} may be obtained.

In general

(22) $$l_x^{ii} = l_{x-1}^{ii}(1 - q_{x-1}^i) + p_{x-1}^{ai} \cdot l_{x-1}^{aa}$$

and

(23) $$l_x^{aa} = l_x - l_x^{ii}.$$

The process is repeated until the table is completed.

When the function p_x^{ai} is used in the construction of a Combined Mortality and Disability table, it is usual to tabulate under the heading "Number Becoming Disabled in Course of the Year" only those who become disabled during the year and are alive at the end of the year. This number for any age x is the product of p_x^{ai} and l_x^{aa}.

If a column of "Deaths among Active Lives" is shown, it would comprise for each age all deaths during the year among lives active at the beginning of the year. A life active at the beginning of the year becoming disabled and subsequently dying within the year would be included among the deaths among

lives active at the beginning of the year. We have used the symbol d_x^{aa} for the number dying while active between ages "x" and "$x + 1$." The symbol d_x^a may properly be used for the deaths between ages "x" and "$x + 1$" among lives active at age x. The column d_x^a may be completed from the relation

$$d_x^a = d_x - l_x^{ii} q_x^i.$$

Students who refer to Hunter's Table on page 66 of Volume XII, T.A.S.A., should notice that the values shown in the column headed "Probability of Dying in Active State" are equal at any age to $\frac{d_x^a}{l_x^{aa}}$. They are really values of q_x^a where q_x^a is the probability of a life active at age x dying within the year. Since Hunter's Table was published, the Actuarial Society have adopted an official disability notation, and to avoid confusion the student should make the following changes in the headings of Hunter's Table:

r_x should be changed to p_x^{ai},
q_x^{aa} should be changed to q_x^a,
$l_x^{aa} r_x$ should be changed to $l_x^{aa} p_x^{ai}$.

Hunter's Table is published in a convenient form for most purposes. That table shows values of $l_x^{aa} \cdot p_x^{ai}$ and q_x^a, and if it is desired for any purpose to obtain a table giving values of i_x, d_x^{ii}, r_x and d_x^{aa} it may readily be done by the use of the following formulæ. On the assumption that disability occurs, on the average, in the middle of the year,

$$l_x^{aa} \cdot p_x^{ai} = i_x(1 - \tfrac{1}{2} q_x^i),$$

$$i_x = \frac{l_x^{aa} \cdot p_x^{ai}}{1 - \tfrac{1}{2} q_x^i}, \tag{24}$$

$$d_x^{ii} = (l_x^{ii} + \tfrac{1}{2} i_x) q_x^i, \tag{25}$$

$$r_x = \frac{i_x}{l_x^{aa}} = \frac{p_x^{ai}}{1 - \tfrac{1}{2} q_x^i}, \tag{26}$$

$$d_x^{aa} = d_x - d_x^{ii}. \tag{26a}$$

At one time there was considerable discussion as to whether there should be a separate table for each age at entry, so that the probability of survivorship or death of an active entrant should be the same as the probability of survivorship according

to the total mortality table; or whether there should be only one table beginning with a single initial age, so that for the succeeding ages the probability of survivorship of the active and disabled lives combined should be in accordance with that of the total mortality table. In the case of a single table the probability of survivorship of an active life is not the same as that in accordance with the total mortality table but is slightly greater on the assumption that the probability of survivorship according to the total table is the probability of survivorship of mixed lives some of whom are disabled. In his paper on page 44 of Volume XII, T. A. S. A., Hunter stated that he had calculated premiums and reserves for the waiver of premium benefit first on the basis of a separate table for each age at entry and then on the basis of a table beginning with a single initial age and that there was no appreciable difference in the results.

It will be recalled that in the construction of a table of mortality and disability, there are three methods of determining the rate of mortality among active lives. The first method described was where an investigation of the mortality among active lives was made and the rates of mortality among active lives directly obtained. An advantage of this method is that there is definitely established at each age the rate of mortality among active lives. If, on this basis, a single table of mortality and disability commencing, for example, at age 15 is subsequently constructed, together with separate tables for each age at entry, it is important to notice that the net premiums and reserves are identical whether the single table or the separate tables are employed.

It is generally stipulated in insurance contracts that benefits dependent upon total and permanent disability will not be granted should disability occur after a fixed age, say 60 or 65. In such cases it is customary, in using any of the formulæ given above, to eliminate the values of r_x, r'_x or $p_x^{a'}$ for ages 60 or 65 and higher ages.

In what follows it will be assumed that the disability benefit becomes payable only in the event of disability occurring before age y, and the formulæ will be based on a single table of mortality and disability commencing at (say) age 15, in which the probability or rate of disability is taken as zero at y and all subsequent ages.

Commutation Columns.

The commutation columns most frequently used in the calculation of premiums and reserves for disability benefits are those based upon l^{aa}, l^{ii}, and l^{i}, respectively.

The student should have clearly in mind the distinction between l_x^{ii} and l_x^{i}. The number of disabled persons living at age x according to a combined mortality and disability table is represented by l_x^{ii}, whereas the number of persons living at age x out of a certain number of disabled lives selected as a radix at the initial age of a table of disabled life mortality is represented by l_x^{i}. Commutation columns are formed in the usual way, as follows:—

$$D_x^{aa} = v^x l_x^{aa}, \qquad C_x^{aa} = v^{x+1} d_x^{aa},$$
$$N_x^{aa} = \sum_{n=0}^{n=\omega-x} D_{x+n}^{aa}, \qquad M_x^{aa} = \sum_{n=0}^{n=\omega-x} C_{x+n}^{aa},$$
$$S_x^{aa} = \sum_{n=0}^{n=\omega-x} N_{x+n}^{aa}, \qquad R_x^{aa} = \sum_{n=0}^{n=\omega-x} M_{x+n}^{aa},$$
$$D_x^{ii} = v^x l_x^{ii}, \qquad C_x^{ii} = v^{x+1} d_x^{ii},$$
$$N_x^{ii} = \sum_{n=0}^{n=\omega-x} D_{x+n}^{ii}, \qquad M_x^{ii} = \sum_{n=0}^{n=\omega-x} C_{x+n}^{ii},$$
$$S_x^{ii} = \sum_{n=0}^{n=\omega-x} N_{x+n}^{ii}, \qquad R_x^{ii} = \sum_{n=0}^{n=\omega-x} M_{x+n}^{ii},$$
$$D_x^{i} = v^x l_x^{i}, \qquad C_x^{i} = v^{x+1} d_x^{i},$$
$$N_x^{i} = \sum_{n=0}^{n=\omega-x} D_{x+n}^{i}, \qquad M_x^{i} = \sum_{n=0}^{n=\omega-x} C_{x+n}^{i},$$
$$S_x^{i} = \sum_{n=0}^{n=\omega-x} N_{x+n}^{i}, \qquad R_x^{i} = \sum_{n=0}^{n=\omega-x} M_{x+n}^{i}.$$

Annuities.

The notation and the formulæ for annuities will now be given.

a_x^{aa} = the value of an annuity-due on an active life payable during activity.

a_x^{i} = the value of an annuity-due on a disabled life.

a_x^{a} = the value of an annuity-due on an active life payable during survival.

a_x^{ai} = the value of an annuity payable at the end of each year provided an active life now aged x is then alive but disabled.

$$(27)\quad \mathrm{a}_x^{aa} = \frac{N_x^{aa}}{D_x^{aa}}, \qquad (28)\quad \mathrm{a}_{x\overline{n}|}^{aa} = \frac{N_x^{aa} - N_{x+n}^{aa}}{D_x^{aa}},$$

$$(29)\quad \mathrm{a}_x^{i} = \frac{N_x^{i}}{D_x^{i}}, \qquad (30)\quad \mathrm{a}_{x\overline{n}|}^{i} = \frac{N_x^{i} - N_{x+n}^{i}}{D_x^{i}}.$$

Of the l_x^{ii} disabled persons living at age x according to a combined mortality and disability table there will be ${}_np_x^{i} \cdot l_x^{ii}$ alive n years hence; therefore, of the l_x^{aa} active persons living

at age x there will be $l_{x+n} - {}_np_x^i \cdot l_x^{ii}$ persons alive n years hence. Where ${}_np_x^a$ represents the probability that an active life age x will survive n years,

$$ {}_np_x^a = \frac{l_{x+n} - {}_np_x^i \cdot l_x^{ii}}{l_x^{aa}}, $$

(31) $$ {}_np_x^a = \frac{l_x}{l_x^{aa}} {}_np_x - \frac{l_x^{ii}}{l_x^{aa}} \cdot {}_np_x^i. $$

Hence

$$ \Sigma v^n \cdot {}_np_x^a = \frac{l_x}{l_x^{aa}} \Sigma v^n {}_np_x - \frac{l_x^{ii}}{l_x^{aa}} \Sigma v^n {}_np_x^i, $$

(32) $$ \mathbf{a}_x^a = \frac{l_x}{l_x^{aa}} \cdot \mathbf{a}_x - \frac{l_x^{ii}}{l_x^{aa}} \cdot \mathbf{a}_x^i, $$

and

(33) $$ \mathbf{a}_{\overline{xn}|}^a = \frac{l_x}{l_x^{aa}} \mathbf{a}_{\overline{xn}|} - \frac{l_x^{ii}}{l_x^{aa}} \cdot \mathbf{a}_{\overline{xn}|}^i. $$

By definition

(34) $$ a_x^{ai} = \mathbf{a}_x^a - \mathbf{a}_x^{aa}, $$

and

(35) $$ a_{\overline{xn-1}|}^{ai} = \mathbf{a}_{\overline{xn}|}^a - \mathbf{a}_{\overline{xn}|}^{aa}, $$

hence

(36) $$ a_x^{ai} = \frac{l_x}{l_x^{aa}} \mathbf{a}_x - \frac{l_x^{ii}}{l_x^{aa}} \mathbf{a}_x^i - \mathbf{a}_x^{aa}, $$

(37) $$ a_x^{ai} = \mathbf{a}_x - \mathbf{a}_x^{aa} + \frac{l_x^{ii}}{l_x^{aa}} (\mathbf{a}_x - \mathbf{a}_x^i), $$

and

(38) $$ a_{\overline{xn-1}|}^{ai} = \mathbf{a}_{\overline{xn}|} - \mathbf{a}_{\overline{xn}|}^{aa} + \frac{l_x^{ii}}{l_x^{aa}} (\mathbf{a}_{\overline{xn}|} - \mathbf{a}_{\overline{xn}|}^i). $$

The value of a_x^{ai} as expressed by equation (36) may be deduced directly by first principles. If l_x lives have an annuity of 1 per annum ceasing at death, they will receive in the aggregate each year the same amount as l_x^{aa} active lives and l_x^{ii} disabled lives receiving similar annuities. In symbols this would be

$$ l_x \mathbf{a}_x = l_x^{aa} \mathbf{a}_x^a + l_x^{ii} \mathbf{a}_x^i; $$

hence

$$ l_x^{aa} \mathbf{a}_x^a = l_x \mathbf{a}_x - l_x^{ii} \mathbf{a}_x^i; $$

but as

$$ \mathbf{a}_x^a = \mathbf{a}_x^{aa} + a_x^{ai}, $$

$$ l_x^{aa}(\mathbf{a}_x^{aa} + a_x^{ai}) = l_x \mathbf{a}_x - l_x^{ii} \mathbf{a}_x^i, $$

$$ l_x^{aa} a_x^{ai} = l_x \mathbf{a}_x - l_x^{aa} \mathbf{a}_x^{aa} - l_x^{ii} \mathbf{a}_x^i, $$

$$ a_x^{ai} = \frac{l_x}{l_x^{aa}} \mathbf{a}_x - \frac{l_x^{ii}}{l_x^{aa}} \mathbf{a}_x^i - \mathbf{a}_x^{aa}. $$

Very convenient formulæ may be deduced from formula (36) as follows:

$$a_x^{ai} = \frac{l_x}{l_x^{aa}} \cdot \mathfrak{a}_x - \frac{l_x^{ii}}{l_x^{aa}} \cdot \mathfrak{a}_x^i - \mathfrak{a}_x^{aa},$$

$$D_x^{aa} \cdot a_x^{ai} = D_x \mathfrak{a}_x - D_x^{ii} \cdot \mathfrak{a}_x^i - D_x^{aa} \cdot \mathfrak{a}_x^{aa}$$

$$= N_x - D_x^{ii} \mathfrak{a}_x^i - N_x^{aa}.$$

Hence

$$a_x^{ai} = \frac{N_x^{ii} - D_x^{ii} \mathfrak{a}_x^i}{D_x^{aa}} \tag{39}$$

and

$$a_{\overline{xn-1}|}^{ai} = \frac{N_x^{ii} - N_{x+n}^{ii} - D_x^{ii} \mathfrak{a}_{\overline{xn}|}^{i}}{D_x^{aa}}. \tag{40}$$

CALCULATION OF PREMIUMS.

Waiver of Premium Benefit.

An example of this benefit appears on page 4.

As has been previously stated when a premium is waived the amount of the waived premium is not charged against the policy, but the policy and all guarantees in it continue precisely as though premiums had been paid each year. This benefit is usually limited to the occurrence of total and permanent disability before the insured attains the age of 60 or 65.

The formulae that follow apply to the benefit of waiving either the gross or net premium P, in the event of disability occurring before age y, where premiums for the benefit cease at age y. In practice the combined mortality and disability table is adjusted so that beyond y the rate of disability is zero, and it is therefore assumed in developing the following formulæ that this adjustment has been made.

a_x^{ai} calculated on the basis of a combined mortality and disability table that has been adjusted in the manner described in the preceding paragraph therefore represents the present value of one per annum payable during the disability of a life aged x, now active, provided disability occurs before age y.

Whole Life Policy.—At the inception of the contract the value of the waiver of premium benefit is

$$P \cdot a_x^{ai}, \tag{41}$$

and remembering that premiums for the waiver of premium

benefit cease at age y, the annual premium for the benefit is

$$P \cdot a_x^{ai} \cdot \frac{D_x^{aa}}{N_x^{aa} - N_y^{aa}} \quad \text{or} \quad \frac{P \cdot a_x^{ai}}{\mathrm{a}_{\overline{xy-x}|}^{aa}}. \tag{42}$$

Limited Payment Life and Endowment Policy.—At the inception of the contract the value of the waiver of premium benefit is

$$P \cdot a_{\overline{xn-1}|}^{ai}, \tag{43}$$

where n is the number of payments under a limited-payment life policy, or the term of the endowment (only an n-payment n-year endowment being considered). Where $x + n =$ or $< y$ the premium for the waiver of premium benefit is payable for n years and the net annual premium is

$$\frac{P \cdot a_{\overline{xn-1}|}^{ai}}{\mathrm{a}_{\overline{xn}|}^{aa}}, \tag{44}$$

but where $x + n > y$ the premium for the benefit ceases at age y and the net annual premium for the benefit is

$$\frac{P \cdot a_{\overline{xn-1}|}^{ai}}{\mathrm{a}_{\overline{xy-x}|}^{aa}}. \tag{45}$$

Whole Life Policy.—Using formula (39) the net annual premium for the waiver of premium benefit is

$$\frac{P(N_x^{ii} - D_x^{ii}\mathrm{a}_x^{i})}{N_x^{aa} - N_y^{aa}}. \tag{46}$$

Limited Payment Life and Endowment Policy.—Using formula (40) the net annual premium for the waiver of premium benefit, where $x + n =$ or $< y$,

$$\frac{P \cdot (N_x^{ii} - N_{x+n}^{ii} - D_x^{ii}\mathrm{a}_{\overline{xn}|}^{i})}{N_x^{aa} - N_{x+n}^{aa}}, \tag{47}$$

or where $x + n > y$,

$$\frac{P \cdot (N_x^{ii} - N_{x+n}^{ii} - D_x^{ii}\mathrm{a}_{\overline{xn}|}^{i})}{N_x^{aa} - N_y^{aa}}. \tag{48}$$

Annuity During Disability.

An example of this type of disability benefit appears on page 6. Under this benefit the payments during disability do not

affect the payment of the face amount payable at death or maturity. The formulæ that follow are applicable to the case where the first payment is deferred until one year after the receipt of proof of disability, and subsequent payments are made annually thereafter.

k represents the annual annuity payment.

n represents the number of annual premiums under a limited-payment life policy or the term of an endowment policy—only an n-payment n-year endowment policy being considered.

Whole Life Policy.—At the inception of the contract the value of the benefit is

$$\frac{1}{D_x^{aa}} \cdot k \sum_{s=z}^{s=y-1} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}}^{i} \tag{49}$$

and as the annual premium for the benefit is payable until age y the net annual premium is

$$\frac{k}{N_x^{aa} - N_y^{aa}} \sum_{s=z}^{s=y-1} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}}^{i}. \tag{50}$$

Limited Payment Life Policy.—At the inception of the contract the value of the benefit is precisely the same as under a whole life policy, and where $x + n =$ or $> y$ the net annual premium for the benefit is also precisely the same as that under a whole life policy. Where $x + n < y$ the net annual premium for the benefit is

$$\frac{k}{N_x^{aa} - N_{x+n}^{aa}} \sum_{s=z}^{s=y-1} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}}^{i}. \tag{51}$$

Endowment Policy.—Where $x + n =$ or $< y$ the annual premium for the disability benefit is payable for the entire term of the endowment, therefore the net annual premium for the benefit is

$$\frac{k}{N_x^{aa} - N_{x+n}^{aa}} \sum_{s=z}^{s=x+n-2} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}:\overline{x+n-s-1}|}^{i}. \tag{52}$$

Where $x + n > y$ the annual premium for the disability benefit ceases at age y and the benefit is not received unless disability occurs before age y. The net annual premium for the benefit is

$$\frac{k}{N_x^{aa} - N_y^{aa}} \sum_{s=z}^{s=y-1} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}:\overline{x+n-s-1}|}^{i}. \tag{53}$$

The above are the formulæ in the case where the first payment of the disability annuity is deferred until one year after the receipt of proof of disability. If it is desired to use the above formulæ in connection with Hunter's table, it should be noticed that they include the factor r_x, whereas Hunter's Table gives only the values of p_x^{ai}. According to formula (26)

$$r_x = \frac{p_x^{ai}}{1 - \frac{1}{2}q_x^i};$$

hence, as the values of p_x^{ai} and q_x^i are given, a column of r_x may be constructed.

For convenience in calculating the premiums it may be assumed that the first payment of the disability annuity is payable on the policy anniversary following commencement of the disability, and subsequent payments yearly thereafter. In the case of an endowment the last payment would be one year before the end of the period. Such an assumption is on the side of safety. It enables us to use Hunter's Table without modification.

Formula (49) would then become

$$\frac{k}{D_x^{aa}} \sum_{s=x}^{s=y-1} D_s^{aa} p_s^{ai} v a_{s+1}^i \quad \text{or} \quad k a_x^{ai}. \qquad \text{(see No. 41)}$$

Formula (53) would become

$$\frac{k}{N_x^{aa} - N_y^{aa}} \sum_{s=x}^{s=y-1} D_s^{aa} p_s^{ai} v a_{s+1:\overline{x+n-s-1}|}^i \quad \text{or} \quad \frac{k a_{x:\overline{n-1}|}^{ai}}{a_{x:\overline{y-x}|}^{aa}}$$

(see No. 45).

The other formulæ would be similarly modified.

It is impossible to give within the limits of a single study all the formulæ for every benefit that has been given by the companies including the modifications for the different provisions as to the date of the first disability payment.

Payment may be made

(*a*) One year after receipt of proof;
(*b*) Six months after receipt of proof;
(*c*) One year after the anniversary next succeeding receipt of proof.

If, however, the student will study and master the methods outlined here, he will be able to solve with comparatively little trouble the problems which will present themselves in practice.

PAYMENT OF FACE AMOUNT IN INSTALMENTS DURING DISABILITY. BALANCE, IF ANY, PAYABLE AT DEATH OR MATURITY.

An example of this type of disability benefit appears on page 5. Such clauses usually provide for the payment of the face amount in ten or twenty annual instalments, the first instalment to be made usually one year after receipt of proof of disability or one year after the anniversary of the policy next succeeding the receipt of proof of disability. Each instalment paid reduces the face amount of the policy by the amount of the instalment, so that the amount payable at death or maturity is the difference between the face of the policy and the amount paid in instalments. When the face amount of the policy has been paid in instalments no further instalments are due.

The following formulae are applicable to the case where the first instalment becomes due one year after the anniversary next succeeding the receipt of proof of total and permanent disability. The symbols used may be defined thus:

k represents the number of instalments, $1/k$ being the amount of each instalment,

n represents the number of years' premium which are payable under a limited payment life policy, or the term of an endowment policy (only an n-payment n-year endowment policy being considered),

d represents the discount on 1 payable one year hence.

Whole Life Policy.—By paying a part of the policy before death the company loses interest on the instalment paid from the date of payment to the date of death, and the value of that interest is measured by the mortality among disabled lives.

In the case of a life aged $(z + 1)$ which has become disabled during the preceding year an instalment of $1/k$ will be paid on attaining age $(z + 2)$. The value of the interest lost by making this advance payment is an annuity-due of d/k so that the value of the benefit, on account of the first instalment, is

$$\frac{d}{k} \cdot \frac{N^i_{z+2}}{D^i_{z+1}}.$$

Similarly, the value of the benefit at the end of the year in

which disability occurs on account of the second advance payment is

$$\frac{d}{k} \cdot \frac{N^i_{x+2}}{D^i_{x+1}}.$$

As there are k possible instalments the value at the end of the year in which disability occurs on account of the prepayment of all possible instalments is

$$\frac{d}{k} \cdot \frac{1}{D^i_{x+1}} \{N^i_{x+2} + N^i_{x+3} + \cdots + N^i_{x+k+1}\},$$

$$= \frac{d}{k}\left(\frac{S^i_{x+2} - S^i_{x+k+2}}{D^i_{x+1}}\right), \tag{54}$$

and at the inception of the contract the value of the benefit is

$$\frac{1}{D^{aa}_x} \sum_{x}^{y-1} D^{aa}_x \cdot p^{ai}_x \cdot v \cdot \frac{d}{k} \cdot \left(\frac{S^i_{x+2} - S^i_{x+k+2}}{D^i_{x+1}}\right). \tag{55}$$

As premiums for the benefit are payable until age y the net annual premium for the benefit is

$$\frac{1}{N^{aa}_x - N^{aa}_y} \sum_{x}^{y-1} D^{aa}_x \cdot p^{ai}_x \cdot v \cdot \frac{d}{k} \cdot \left(\frac{S^i_{x+2} - S^i_{x+k+2}}{D^i_{x+1}}\right). \tag{56}$$

Limited Payment Life Policy.—At the beginning of the contract the value of this benefit is the same as that under a whole life policy issued at the same age and when $x + n =$ or $> y$ the annual premium for the benefit is also the same as under a whole life policy, because in this case the annual premium for the benefit will cease at age y. Where $x + n < y$ the net annual premium for the benefit is

$$\frac{1}{N^{aa}_x - N^{aa}_{x+n}} \sum_{x}^{y-1} D^{aa}_x \cdot p^{ai}_x \cdot v \cdot \frac{d}{k}\left(\frac{S^i_{x+2} - S^i_{x+k+2}}{D^i_{x+1}}\right). \tag{57}$$

Endowment Policy.—Under an endowment policy the temporary annuity

$$\frac{N^i_x - N^i_{x+n}}{D^i_x}$$

measures the value of the interest from the date when an instalment is paid until the date of death or maturity. Furthermore, the payment of instalments may be terminated either by death

or maturity, so that the value of the benefit at the end of the year in which disability occurs, the insured's age then being $z+1$, is

$$\begin{aligned}(58)\quad &\frac{d}{k}\left\{\frac{D^i_{z+2}}{D^i_{z+1}}\left(\frac{N^i_{z+2}-N^i_{x+n}}{D^i_{z+2}}\right)+\frac{D^i_{z+3}}{D^i_{z+1}}\left(\frac{N^i_{z+3}-N^i_{x+n}}{D^i_{z+3}}\right)\right.\\ &\qquad\left.+\cdots+\frac{D^i_{z+m}}{D^i_{z+1}}\left(\frac{N^i_{z+m}-N^i_{x+n}}{D^i_{z+m}}\right)\right\}\\ &=\frac{d}{k}\cdot\frac{1}{D^i_{z+1}}\{N^i_{z+2}+N^i_{z+3}+\cdots+N^i_{z+m}-(m-1)N^i_{x+n}\}\\ &=\frac{d}{k}\cdot\left\{\frac{S^i_{z+2}-S^i_{z+m+1}-(m-1)N^i_{x+n}}{D^i_{z+1}}\right\},\end{aligned}$$

where m equals $k+1$ or $x+n-z-1$, whichever is the less.

Where $x+n=$ or $<y$ the annual premium for the benefit is payable for n years, and the net annual premium for the benefit is

$$(59)\quad \frac{1}{N^{aa}_x-N^{aa}_{x+n}}\left\{\sum_{z=x}^{z=x+n-3}D^{aa}_z\cdot p^{ai}_z\cdot v\cdot\frac{d}{k}\times\left[\frac{S^i_{z+2}-S^i_{z+m+1}-(m-1)N^i_{x+n}}{D^i_{z+1}}\right]\right\},$$

but where $x+n>y$ the annual premium for the benefit ceases at age y and the benefit is not received unless disability occurs before age y and the premium for the benefit is

$$(60)\quad \frac{1}{N^{aa}_x-N^{aa}_y}\left\{\sum_{z=x}^{z=y-1}D^{aa}_z\cdot p^{ai}_z\cdot v\cdot\frac{d}{k}\times\left[\frac{S^i_{z+2}-S^i_{z+m+1}-(m-1)N^i_{x+n}}{D^i_{z+1}}\right]\right\}.$$

As the benefit ceases unless disability occurs at least two years prior to maturity, the summation in equation (60) must cease when $z=x+n-3$. Accordingly when $x+n=y+1$ the summation is $\sum_{z=x}^{z=y-2}$ instead of $\sum_{z=x}^{z=y-1}$, the formula remaining otherwise the same.

When a policy provides for an annuity or for the payment of the face amount in instalments during disability, it generally also provides for waiver of premiums during disability. The total premium for the disability benefits will then be the sum of the premium for the waiver benefit and the premium for the

annuity or instalment benefit. In calculating the premium for the waiver benefit the premium to be waived is considered to be the premium for the regular insurance exclusive of the disability benefit, because the premium for the disability benefit is calculated on the assumption that it will cease in event of disability.

The waiver of premium benefit contained in a limited payment life or an endowment policy ceases when the last premium is paid. The benefit considered on pages 41 to 43 ceases unless disability occurs prior to one year before the maturity of an endowment policy; the benefit considered on pages 44 to 46 ceases unless disability occurs prior to two years before the maturity of an endowment policy. Where $x + n =$ or $< y$ premiums for these benefits are calculated on the assumption that they will be received for n years, unless the insured dies or becomes disabled before that time. In other words it is assumed that premiums will be received after the benefit ceases. Although many policies grant the insured the privilege of canceling at any time the disability feature and reducing the premium by the amount of the premium for the disability benefits, yet it is not believed that many policyholders will take advantage of this condition.

RESERVES ON ACTIVE LIVES.

Under disability benefits designated on page 2, numbers 1, 2 and 3, the reserve for the disability benefit may be determined from the general formula:

$$\left(\begin{matrix}\text{Present value}\\ \text{of benefit}\end{matrix}\right) \text{ minus } \left(\begin{matrix}\text{Present value of}\\ \text{future net premiums}\end{matrix}\right).$$

This reserve is, of course, in addition to the regular reserve for the policy. When two or more disability benefits are combined in the same policy the reserve to be held for those disability benefits will be the total of the reserves under all the disability benefits.

The formulæ now to be given for reserves on active lives do not have any meaning when a duration of the policy is reached which makes it impossible for an active life to become entitled to the disability benefit which is dealt with by the formula. The terminal reserve in such cases should be taken as zero.

The Insurance Departments of New York and of Massachusetts have adopted tables of reserves based upon the tables constructed by Hunter. In constructing these tables for ordinary life policies with benefit limited to disability occurring prior to age y, the reserves on active lives were calculated by the formula

$$\pi_x \left[(a_{x+t} - a^{aa}_{x+t}) + \frac{D^{ii}_{x+t}}{D^{aa}_{x+t}} (a_{x+t} - a^{i}_{x+t}) \right] - P^d \cdot a^{aa}_{x+t:\overline{y-x-t}|},$$

where

$$P^d = \frac{\pi_x (N^{ii}_x - D^{ii}_x a^{i}_x)}{N^{aa}_x - N^{aa}_y}$$

(see formula 46).

It will be noticed that the premium assumed to be waived is π_x, which is the net premium according to the American Table of Mortality.

It was found in many cases that the terminal reserves were negative, but it was not deemed advisable to take account of them in practice. Theoretically when the terminal reserve is negative, one half of a net one-year-term disability premium for the attained age should be held as a mean reserve. In practice, however, the use of one half of the net disability premium for the policy is simpler and produces satisfactory results in the aggregate. This method was adopted by the Insurance Departments.

It will be a useful exercise for the student to show that the reserve formula used by the Insurance Departments is equal to

$$\frac{\pi_x (N^{ii}_{x+t} - D^{ii}_{x+t} a^{i}_{x+t})}{D^{aa}_{x+t}} - \frac{P^d (N^{aa}_{x+t} - N^{aa}_{60})}{D^{aa}_{x+t}}.$$

This is formula (61) on page 49, if P is given the value π_x and y is given the value 60.

Waiver of Premium Benefit.

Let P be the annual premium that is to be waived. It may be either the gross or the net premium for the policy without the disability benefits.

Let P^d be the net annual premium for the waiver of premium benefit.

Let n be the number of years premiums are payable under a limited payment life policy or the term of an endowment policy.

Let t be the duration for which the reserve is desired.

The reserve for the waiver of premium benefit on an active life at the end of t years is as follows:

Whole Life Policy.—

$$(61) \qquad \frac{P(N^{ii}_{x+t} - D^{ii}_{x+t} \cdot \mathbf{a}^{i}_{x+t}) - P^{d}(N^{aa}_{x+t} - N^{aa}_{y})}{D^{aa}_{x+t}}.$$

Limited Payment Life or Endowment Policy.—Where $x + n < y$

$$(62) \qquad \frac{P(N^{ii}_{x+t} - N^{ii}_{x+n} - D^{ii}_{x+t}\mathbf{a}^{i}_{x+t\,\overline{n-t}|}) - P^{d}(N^{aa}_{x+t} - N^{aa}_{x+n})}{D^{aa}_{x+t}}.$$

Where $x + n =$ or $> y$,

$$(63) \qquad \frac{P(N^{ii}_{x+t} - N^{ii}_{x+n} - D^{ii}_{x+t}\mathbf{a}^{i}_{x+t\,\overline{n-t}|}) - P^{d}(N^{aa}_{x+t} - N^{aa}_{y})}{D^{aa}_{x+t}}.$$

The reserve for the waiver of premium benefit will, of course, disappear when the insured attains age y unless he should be disabled before attaining that age. These reserves will frequently be negative and when this happens the company would hold for valuation purposes one half of the net annual premium for the benefit.

Annuity During Disability.

Let P^{d} be the net annual premium for the benefit.

Let k be the annuity, that is the amount payable each year.

Let n be the number of years' premiums which are payable under a limited payment life policy, or the term of an endowment policy.

Let t be the duration for which the reserve for disability benefit is desired.

The reserve on an active life after a policy has been t years in force, where the first payment of the annuity is made one year after the receipt of proof of disability, will be as follows:

Ordinary Life Policy.—

$$(64) \qquad \frac{k(\sum_{z=x+t}^{z=y-1} D^{aa}_{z} \cdot r_{z} \cdot v^{\frac{1}{2}} \cdot a^{i}_{z+\frac{1}{2}}) - P^{d}(N^{aa}_{x+t} - N^{aa}_{y})}{D^{aa}_{x+t}}.$$

Limited Payment Life Policy.—When $x + n =$ or $> y$ the premium and the benefit are precisely the same as under a whole life policy, consequently the reserve is the same, but when

$x + n < y$ the reserve is

$$(65) \qquad \frac{k(\sum_{s=x+t}^{s=y-1} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}}^{i}) - P^d(N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}.$$

When t is equal to or greater than n the latter part of this equation vanishes.

Endowment Policy.—Where $x + n =$ or $< y$,

$$(66) \qquad \frac{k(\sum_{s=x+t}^{s=x+n-2} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}\cdot\overline{x+n-s-1}|}^{i}) - P^d(N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}.$$

Where $x + n > y$,

$$(67) \qquad \frac{k(\sum_{s=x+t}^{s=y-1} D_s^{aa} \cdot r_s \cdot v^{\frac{1}{2}} \cdot a_{s+\frac{1}{2}\cdot\overline{x+n-s-1}|}^{i}) - P^d(N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}.$$

PAYMENT OF FACE AMOUNT IN INSTALMENTS; BALANCE, IF ANY, AT DEATH OR MATURITY.

The following formulæ are applicable when the first instalment becomes due one year after the anniversary next succeeding the receipt of proof of disability.

Let k represent the number of annual instalments, $1/k$ being the amount of each instalment.

Let P^d represent the net annual premium for the benefit.

Let n be the number of years' premiums payable under a limited payment life policy or the term of an endowment policy, an n-payment n-year endowment only being considered.

Let d represent the discount on 1 payable one year hence, and

Let t be the duration for which the reserve is desired.

The reserve on an active life at the end of the tth policy year is as follows:

Whole Life Policy.—

$$(68) \qquad \frac{\left[\sum_{s=x+t}^{s=y-1} D_s^{aa} \cdot p_s^{ai} \cdot v \cdot \frac{d}{k} \cdot \left(\frac{S_{s+2}^{i} - S_{s+k+2}^{i}}{D_{s+1}^{i}}\right)\right] - P^d(N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}.$$

Limited Payment Life Policy.—When $x + n =$ or $> y$, the reserve is the same as under a whole life policy; but when

$x + n < y$, the reserve is

$$(69)\quad \frac{\left[\sum_{s=x+t}^{s=y-1} D_s^{aa} \cdot p_s^{ai} \cdot v \cdot \frac{d}{k} \cdot \left(\frac{S_{s+2}^i - S_{s+k+2}^i}{D_{s+1}^i}\right)\right] - P^d(N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}}.$$

Endowment Policy.—When $x + n =$ or $< y$ the reserve is

$$(70)\quad \frac{\left[\sum_{s=x+t}^{s=x+n-3} D_s^{aa} \cdot p_s^{ai} \cdot v \cdot \frac{d}{k} \cdot \left(\frac{S_{s+2}^i - S_{s+m+1}^i - (m-1)N_{x+n}^i}{D_{s+1}^i}\right)\right] - P^d(N_{x+t}^{aa} - N_{x+n}^{aa})}{D_{x+t}^{aa}},$$

where $m = (k + 1)$ or $(x + n - z - 1)$ whichever is less. When $x + n > y$, the reserve is

$$(71)\quad \frac{\left[\sum_{s=x+t}^{s=y-1} D_s^{aa} \cdot p_s^{ai} \cdot v \cdot \frac{d}{k}\left(\frac{S_{s+2}^i - S_{s+m+1}^i - (m-1)N_{x+n}^i}{D_{s+1}^i}\right)\right] - P^d(N_{x+t}^{aa} - N_y^{aa})}{D_{x+t}^{aa}}.$$

Where $x + n = y + 1$ the summation in formula (71) ceases with the term $z = x + n - 3$.

RESERVES ON DISABLED LIVES.

Waiver of Premium Benefit.

In order to enable the student to readily understand the principle underlying the nature of the reserve to be carried on a disabled life, the following demonstration has been taken from T. A. S. A., XII, 63.

"For clearness of analysis, let us imagine that there are three separate and independent companies carrying on the successive stages of this kind of insurance, as follows:—

Company A, which is a plain life insurance company and has nothing to do with the "waiver of premium" feature;

Company B, which guarantees to each person when he insures in Company A that if he shall thereafter become disabled it will purchase for him in Company C an annuity equal to the net premium which he pays to Company A;

Company C, which, whenever an insured in Company A becomes disabled, receives from Company B the price of the annuity.

We first ask the question: What reserve should Company C hold on the disabled lives which compose its entire membership?

It is evident that the reserve it should hold on any such life must be equal to the single premium for the annuity, upon the disabled life mortality table.

Next comes the question: What reserve should Company B hold on its membership? It would seem that such reserve should be the present value of a risk equal to the single premium which it would have to pay to Company C in case the life should become disabled in any subsequent year, less the present value of the net premium it must charge the insured for this indemnity." (This is equivalent to the reserve for active lives as given by formula 61).

The reserve to be held by Company A is the regular insurance reserve only, i.e., American 3 per cent, Actuaries 4 per cent, or whatever reserve is generally carried.

It is evident that, for lives that have become disabled, the proper reserve is the regular insurance reserve, say, American 3½ per cent, plus the present value of future premiums valued by an annuity on the basis of disabled life mortality.

Let P represent the premium that is to be waived. This premium will be the gross or the net premium for the policy exclusive of the premium for the disability benefits;

Let t represent the number of years the policy has been in force. Let n represent the number of payments under a limited-payment life policy or the term of an endowment policy;

Then under an ordinary life policy, assuming notice of disability to be received immediately before the policy anniversary, the disability reserve when a life is disabled is

$$P \cdot \mathbf{a}^{i}_{x+t}, \tag{72}$$

and under a limited-payment life or endowment policy the disability reserve is

$$P \cdot \mathbf{a}^{i}_{x+t\overline{n-t}|}. \tag{73}$$

Annuity During Disability.

The liability of the company with reference to this benefit when a life is disabled is evidently the present value of the annuity calculated on the basis of disabled life mortality. The formulæ that follow are applicable when the first payment is deferred until one year after receipt of proof of disability and the

annuity payments on a disabled life will therefore be made on the average in the middle of the policy year.

Let k represent the annuity.

Let t represent the integral number of years the policy has been in force.

Let n represent the number of payments under a limited-payment life policy, or the term of an endowment policy.

Under an ordinary life or limited-payment life policy the disability reserve for this benefit will be

$$k \times {}_{\frac{1}{2}|}\mathbf{a}^{i}_{x+t}, \tag{74}$$

and under an endowment policy the disability reserve for this benefit will be

$$k \times {}_{\frac{1}{2}|}\mathbf{a}^{i}_{x+t:\overline{n-t}|}. \tag{75}$$

As the annuity benefit is usually accompanied by the waiver of premium benefit the total reserve liability when a life becomes disabled is

(*a*) The regular life insurance reserve for the policy on the assumption that premiums have not been waived;

(*b*) The reserve for the waiver of premium benefit as described on pages 51 and 52.

(*c*) The reserve for the annuity benefit.

Payment of Face Amount in Instalments, Balance, if Any, Payable at Death or Maturity.

Ordinary Life Policy.

We may regard the entire contract, including the waiver of premium benefit, as changed to a series of single-payment endowment assurances on a disabled life and may readily calculate the reserve accordingly.

It is assumed that the first instalment is payable one year after the anniversary next succeeding the receipt of proofs of disability.

If k be the number of instalments payable and $1/k$ the amount of each, and z the age at the beginning of the year in which disability occurs, the total value of the contract at the end of that year would be

$$\frac{1}{k}\left(A^{i}_{z+1:\overline{1}|} + A^{i}_{z+1:\overline{2}|} + \cdots + A^{i}_{z+1:\overline{k}|}\right). \tag{76}$$

From another viewpoint the additional reserve required on account of the payment of instalments in advance would be, in accordance with formula (54),

$$\frac{d}{k} \cdot \frac{S^i_{x+2} - S^i_{x+k+2}}{D^i_{x+1}}. \tag{77}$$

J. B. Gibb (T. A. S. A., XVIII, 123) has shown that the reserve obtained by formula (77), the reserve for the waiver of premium benefit and the regular insurance reserve on the basis of disabled life mortality make up the total value given by formula (76). He draws attention to a practical difficulty by pointing out that in order to comply with the accepted standards of valuation, there must be carried the regular insurance reserve of $1 - (\pi_x + d)\mathrm{a}_{x+n}$ on all risks instead of $1 - (\pi_x + d)\mathrm{a}^a_{x+n}$ on active risks and $1 - (\pi_x + d)\mathrm{a}^i_{x+n}$ on disabled risks. The total reserve on a disabled risk is thus insufficient by the difference between $1 - (\pi_x + d)\mathrm{a}^i_{x+n}$ and $1 - (\pi_x + d)\mathrm{a}_{x+n}$. To provide the instalment or maturity benefit the deficit of $(\pi_x + d)(\mathrm{a}_{x+n} - \mathrm{a}^i_{x+n})$ must be met from the general surplus or from a special fund. If it is to be met from a special fund, Gibb suggests that it might properly be charged to a mortality fund on the ground that the transaction is in the nature of granting a special surrender value in order to get an impaired risk off the books.

It should be remembered that when an insurance company carries a reserve such as the American, the assumption is that this reserve will be sufficient for its business as a whole. As shown by Gibb, the regular American reserve is more than sufficient for lives whose vitality is good but too small for disabled lives. So long as the policies on disabled lives are continued in full force, the sum of the reserves found by treating the active and disabled lives separately will agree with the regular American reserve for the whole company, the deficit in the case of the disabled lives being offset by that carried on the active lives. This follows from the equations

$$l_x = l^{aa}_x + l^{ii}_x$$

$$l_x \cdot \mathrm{a}_x = l^{aa}_x \cdot \mathrm{a}^a_x + l^{ii}_x \mathrm{a}^i_x$$

the reserve for an active life being $1 - (\pi_x + d)\mathrm{a}^a_x$ and that for a disabled life $1 - (\pi_x + d)\mathrm{a}^i_x$. Where, however, the policy is

terminated at disability, this deficit must be charged off, the apparent loss being offset by the excess reserves held on business remaining in force.

As intimated above the regular insurance reserve on a disabled life if taken as $1 - (\pi_x + d)\mathbf{a}^i_{x+n}$ together with the waiver of premium reserve and that found in formula (77) will be exactly sufficient to meet payment of the instalments as they fall due. Whenever an instalment is paid the amount of insurance is reduced by the amount of the instalment. Thus if an instalment of $1/k$ of a unit policy is paid, the regular insurance reserve is reduced in the proportion $1/k$, the premium to be waived becomes $[(k-1)/k]\pi_x$ and the instalment reserve is reduced to the amount necessary to provide for the payment of $(k-1)$ instalments of $1/k$. The total reduction in reserve on account of the payment of the first instalment of $1/k$ at the age of $z+2$ is therefore:—

(*a*) $\frac{1}{k}(A^i_{z+2} - \pi_x \mathbf{a}^i_{z+2})$ = reduction in insurance reserve,

(*b*) $\frac{1}{k}\pi_x \mathbf{a}^i_{z+2}$ = reduction in premium waiver reserve,

(*c*) $\frac{1}{k} d \cdot \mathbf{a}^i_{z+2} = \frac{1}{k} d \cdot \frac{N^i_{z+2}}{D^i_{z+2}}$ = reduction in instalment reserve.

The sum of the values (*a*), (*b*), and (*c*) is $1/k$. We therefore see that when the insurance is reduced in this manner the reserve released is exactly equal in amount to the payment made.

Immediately before payment of the first instalment the additional reserve for disability instalments is

$$\frac{d}{k}\frac{S^i_{z+2} - S^i_{z+k+2}}{D^i_{z+2}},$$

of which there is cancelled after the first instalment is paid,

$$\frac{d}{k}\mathbf{a}^i_{z+2} = \frac{d}{k} \cdot \frac{N^i_{z+2}}{D^i_{z+2}}$$

This leaves the additional reserve just after the first instalment is paid

$$\frac{d}{k} \cdot \frac{S^i_{z+3} - S^i_{z+k+2}}{D^i_{z+2}}.$$

Following this method the additional reserve immediately after payment of the gth instalment would be

$$\frac{d}{k}\frac{S^i_{x+g+2} - S^i_{x+k+2}}{D^i_{x+g+1}}, \tag{78}$$

or, before payment of the $(g + 1)$th,

$$\frac{d}{k} \cdot \frac{S^i_{x+g+2} - S^i_{x+k+2}}{D^i_{x+g+2}}. \tag{79}$$

It may readily be shown that the amount of the reserve released on payment of the $(g + 1)$th instalment, together with the reserve on the insurance of $1/k$ and that on the waiver benefit are exactly sufficient to cover the instalment of $1/k$.

Limited Payment Life Policy.

The additional reserve for disability instalments is the same as for an ordinary life policy.

Endowment Policy.

Let n years be the term of the endowment policy. Let m equal $k + 1$ or $x + n - z - 1$, whichever is the less.

Immediately after payment of the gth instalment the additional reserve for disability instalments will be

$$\frac{d}{k}\left(\frac{S^i_{x+g+2} - S^i_{x+m+1} - (m - g - 1)N^i_{x+n}}{D^i_{x+g+1}}\right). \tag{80}$$

The total reserve liability when disability instalments are payable will be

(*a*) The regular life insurance reserve for the unpaid portion of the policy, on the assumption that premiums have not been waived;

(*b*) The reserve for the future premiums waived, the premiums being reduced accordingly after one or more instalments have been paid;

(*c*) The reserve equal to the present value of the interest on the remaining instalments from the date when each instalment is payable until the death of the insured or the maturity of the policy.

Valuation Tables for Disability Benefits.

The tables used by most companies and by the majority of insurance departments for the valuation of disability benefits are those prepared by Hunter. The basis has been already described, but there are two additional points which should be considered. First, are these tables safe and conservative for premiums and reserves? Second, are the reserves for disabled lives sufficient when the disability has existed for a number of years?

With regard to the first question, the actuaries of several companies stated in 1917 at a public hearing before the insurance department of the State of Wisconsin, that, judging from the experience of their companies, the tables just mentioned were safe and conservative for the calculation of premiums and reserves. In a discussion on disability benefits (T. A. S. A., XIX, 182) the experiences of the Prudential, Travelers and Metropolitan were given. It appears from the data there submitted that the basis of Hunter's tables is conservative, but it should be remembered that the disability benefits granted at the present time are much more liberal than those granted by the Fraternal Orders, on which experience his tables are constructed. It may be that there will be a higher rate of disability because of the greater incentive to fraud, but there are other factors to consider, —namely, the rate of recovery from disability, and the death rate among disabled lives. Through a liberal interpretation of the phrase "total and permanent disability" a distinctly higher ratio of actual to expected claims than 100 per cent. may be experienced; but in that event there will probably be a high rate of recovery from disability, and there may be a shorter period on the average before death than the basic tables indicate. The student should note that the high death rate among disabled lives would not increase the total death rate of the company.

With regard to the second question, the problem can be seen readily by studying the following table, taken from a paper by Mervyn Davis (T. A. S. A. XVII, 229):—